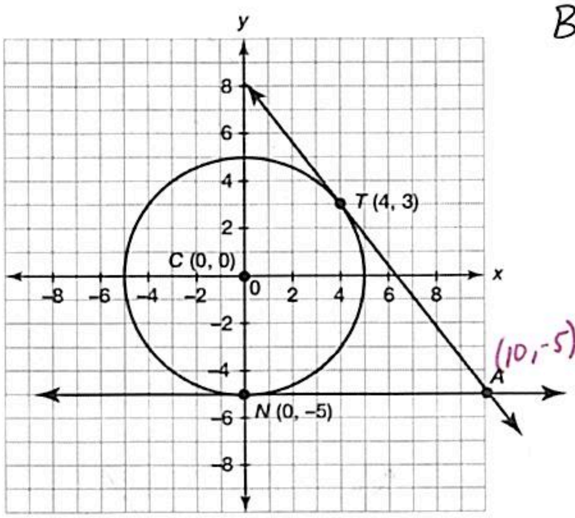
Name: Coach C's Notes

13.1:
Circles and Polygons on the Coordinate Plane

Use the given information to algebraically prove that if two tangents are drawn from the same point on the exterior of the circle, then the tangent segments are congruent.



Between two points → distance formula!

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

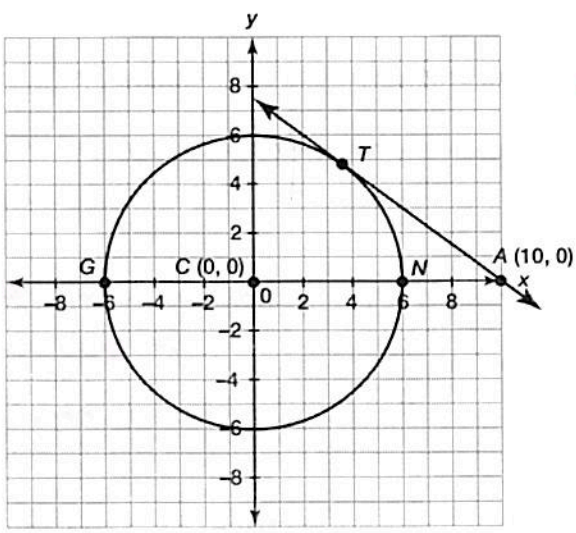
$NA = 10 - 0$

$\boxed{NA = 10}$

$$TA = \sqrt{(10-4)^2 + (-5-3)^2}$$
$$= \sqrt{6^2 + -8^2}$$
$$= \sqrt{36 + 64}$$
$$= \sqrt{100}$$
$$\boxed{TA = 10}$$

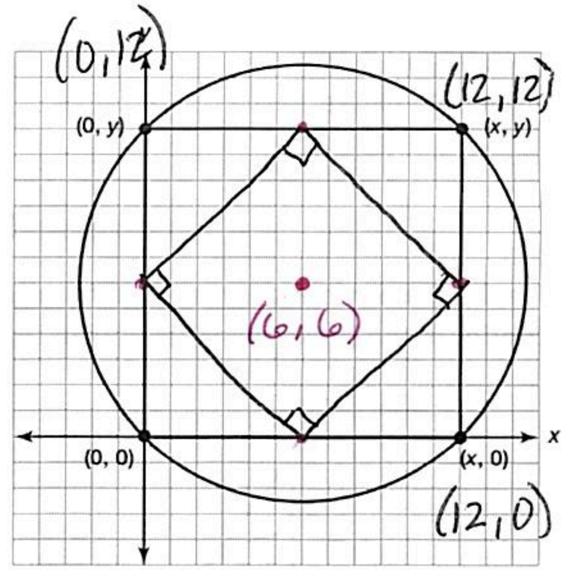
Use the given information to algebraically prove that if a tangent and a secant intersect on the exterior of the circle, then the product of the lengths of the secant segment and its external secant segment is equal to the length of the tangent segment squared.

Line AT is tangent to circle C at point T. Secant $\overline{AG}$ intersects circle C at points N and G. $\overleftrightarrow{AT}$ intersects $\overleftrightarrow{AG}$ at point A. The coordinates of point A are $(10, 0)$.



SKIP

Four points on the circle were connected to form a square.



a. Find the coordinates for each vertex.

b. Find the center of the square.

c. How does the center of the square relate to the circle?

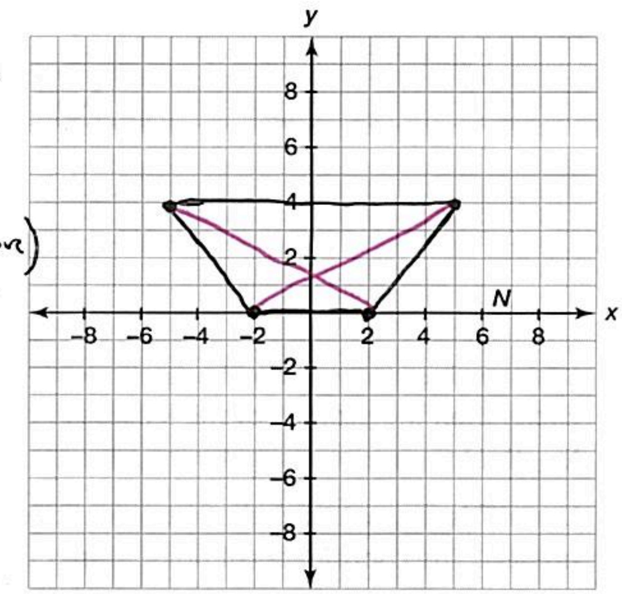
Also center of the circle

d. Find the midpoint of each side of the square.

e. Connect the midpoints of the sides of the square. What polygon is formed?

Another square

Draw a trapezoid using the coordinates $(-5,4)$ $(5,4)$ $(2,0)$ $(-2,0)$.



a. Is this trapezoid isosceles? How do you know?

Yes, the legs are congruent (can use distance formula to prove)

Use the formula $\boxed{y = mx + b}$ when writing the equation for a line.

b. Write an equation for the two diagonals of the trapezoid?

$(-2,0) \to (5,4)$

slope $= \frac{4}{7}$

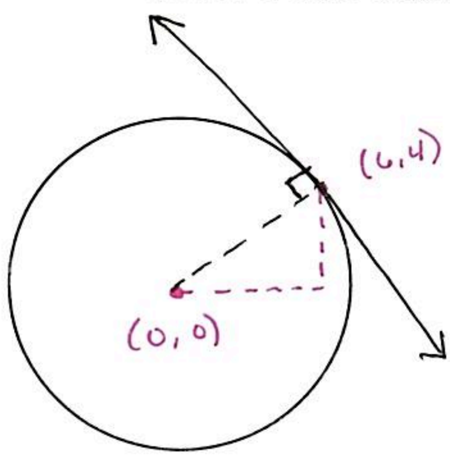
$y = \frac{4}{7}x + 1.5$

$(-5,4) \to (2,0)$

slope $= \frac{-4}{7}$

$y = -\frac{4}{7}x + 1.5$

Determine the equation of a line tangent to circle C with a center at the origin. The point of tangency is $(6,4)$.



Slope of tangent – unknown w/out more information

radius: $(0,0) \to (6,4)$

slope $= \frac{4}{6} \to \frac{2}{3}$

radius and tangent perpendicular

(slope = negative reciprocal

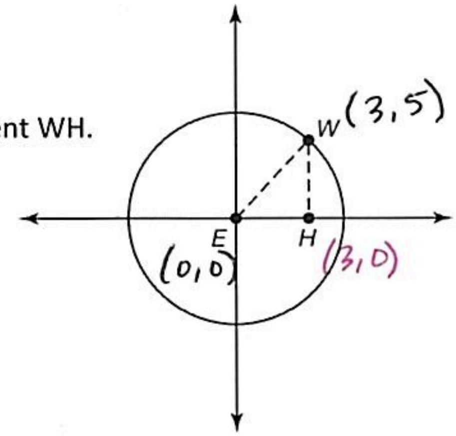
tangent slope $= -\frac{3}{2}$

tangent line

$y = -\frac{3}{2}x + 13$ ← y-int.

13.2:
Deriving the Equation of a Circle

Recall that a circle is the set of points on a plane equidistant from a given point.



Circle E is centered at the origin.

a. The coordinates of point W are (3, 5), determine the length of line segment WH.

5 units

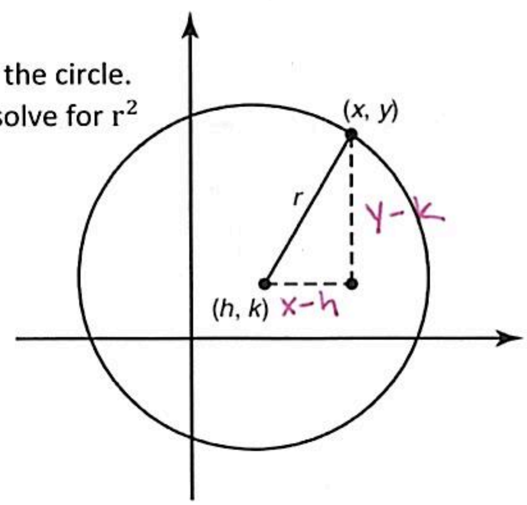
b. Determine the length of line segment EH.

3 units

c. Point W lies on circle E, determine the radius of the circle.

$3^2 + 5^2 = EW^2$

$r = \sqrt{34} \approx 5.83$



Consider a circle with its center located at point (h, k) and a point (x, y) on the circle. Label the sides of the right triangle formed. Use Pythagorean theorem to solve for r^2

$(x-h)^2 + (y-k)^2 = r^2$

The standard form of the equation of a circle centered at (h, k) with radius, r, can be expressed as:

$$\boxed{(x-h)^2 + (y-k)^2 = r^2}$$

* Center: (h, k)

1. Write an equation for a circle with center at the origin and r = 8.

(h, k)
(0, 0)

$(x)^2 + (y)^2 = 8^2$

$\boxed{x^2 + y^2 = 64}$

2. Write an equation for a circle with center (3, -5) and r = 6.

(h, k)
(3, -5)

$(x-3)^2 + (y - -5)^2 = 6^2$

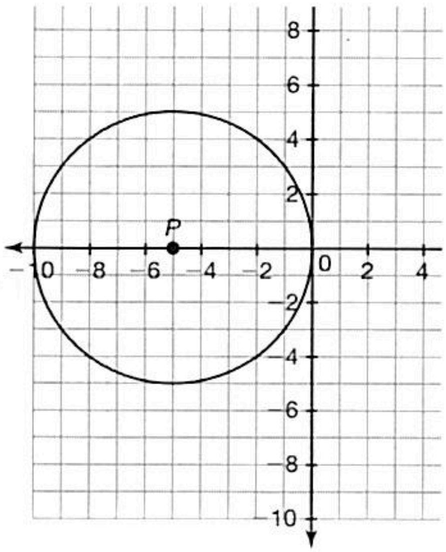
$\boxed{(x-3)^2 + (y+5)^2 = 36}$

3. Write an equation for circle P.

Center $(-5, 0)$

$r = 5$

$(x+5)^2 + y^2 = 25$

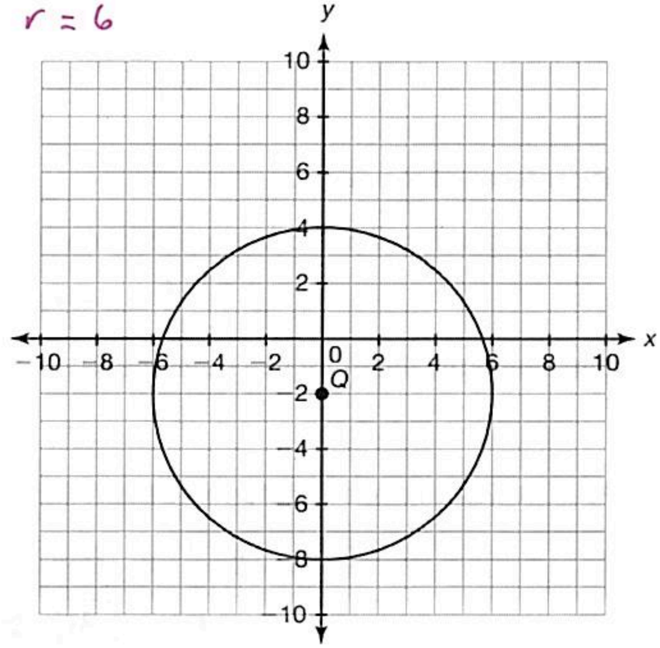


4. Write an equation for circle Q.

Center $(0, -2)$

$r = 6$

$x^2 + (y+2)^2 = 36$



By expanding the binomial terms of the standard form of a circle, $(x-h)^2 + (y-k)^2 = r^2$, the equation can be written as:

$\boxed{(x-h)(x-h)} + \boxed{(y-k)(y-k)} = r^2$

$\boxed{x^2 - xh - xh + h^2} + \boxed{y^2 - ky - ky + k^2} = r^2$

$x^2 + y^2 - 2hx - 2ky + h^2 + k^2 = r^2$

$$\boxed{x^2 + y^2 - 2hx - 2ky + h^2 + k^2 = r^2}$$

The equation for a circle in general form is:

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$
where A, C, D, and E are constants,
$\boxed{A = C,}$ and $x \neq y$

If the coefficient of x^2 and y^2 are not =
then it is not a circle.

In order to identify the center and the radius of a circle written in general form, it is necessary to rewrite the equation in standard form. A mathematical process called completing the square is often necessary to rewrite the equation in standard form.

Complete the square $\longrightarrow$ rewrite in standard form

You can rewrite $x^2 + y^2 - 4x - 6y + 9 = 0$ in standard form by completing the square.

First, use an algebraic transformation to remove the constant term from the variable expression. Write the resulting equation grouping the x-terms together and the y-terms together using sets of parentheses.

$$(x^2 - 4x) + (y^2 - 6y) = -9$$

Next, complete the square within each parenthesis. To do this, examine the first two terms of each quadratic expression. Determine what the constant term would be if each expression were a perfect square trinomial. Add those constant terms to each side of the equation. Write the resulting equation. $\left(\frac{b}{2}\right)^2$

$\frac{-4}{2} = -2^2 = 4$ $(x^2 - \underset{b}{4x} \boxed{+ 4}) + (y^2 - \underset{b}{6y} \boxed{+ 9}) = -9 \boxed{+ 4 + 9}$ $\frac{-6}{2} = -3^2 = 9$

Finally, factor the left side of the equation, which should be a perfect square trinomial. Write the resulting equation.

$$(x - 2)^2 + (y - 3)^2 = 4$$

1. Identify the center point and radius of the circle described by the equation in the worked example.

$(x-2)^2 + (y-3)^2 = 4$ center: $(2, 3)$

$r = 2$

2. Rewrite the following in standard form. $(x^2 \quad) + (y^2 - 4y + 4) = 96 + 4$

a. $x^2 + y^2 - 4y + 4 = 100$
$\qquad -4 \qquad -4$

$x^2 + (y^2 - 4y + 4) = 100$ $\frac{\div 2}{-2}$ square

$\boxed{x^2 + (y-2)^2 = 100}$

b. $x^2 + y^2 - 2x + 4y = 0$

$(x^2 - 2x + 1) + (y^2 + 4y + 4) = 0 + 1 + 4$
$\frac{\div 2}{1} \nearrow 2 \qquad \frac{\div 2}{2} \nearrow 2$

$(x^2 - 2x + 1) + (y^2 + 4y + 4) = 5$

$\boxed{(x-1)^2 + (y+2)^2 = 5}$

c. $x^2 + y^2 + 2x - 4y = 5$

$(x^2 + 2x + 1) + (y^2 - 4y + 4) = 5 + 1 + 4$
$\frac{\div 2}{1} \nearrow 2 \qquad \frac{\div 2}{-2} \nearrow 2$

$(x^2 + 2x + 1) + (y^2 - 4y + 4) = 10 \rightarrow (x+1)^2 + (y-2)^2 = 10$

3. Consider the general form of the equation, $2x^2 + 2y^2 - x + 4y + 2 = 0$
Find the center and radius.

multiplied by 2

$(2x^2 - x) + (2y^2 + 4y) = -2$

$\underline{2}\left(x^2 - \frac{1}{2}x + \frac{1}{16}\right) + \underline{2}\left(y^2 + 2y + 1\right) = -2 \boxed{+\frac{1}{16} + 1}$
$\frac{\div 2}{\frac{1}{4}} \nearrow 2 \qquad \frac{\div 2}{1} \nearrow 2 \qquad + \frac{1}{8} + 2$

center: $\left(\frac{1}{4}, -1\right)$

radius $= \frac{1}{4}$

$\frac{2\left(x - \frac{1}{4}\right)^2}{2} + \frac{2(y+1)^2}{2} = \frac{\frac{1}{8}}{2} \rightarrow \boxed{\left(x - \frac{1}{4}\right)^2 + (y+1)^2 = \frac{1}{16}}$

4. Determine if each equation represents a circle. If so, define the center and radius.

Coefficient same = circle

a. $x^2 + y^2 - 2x + 4y + 4 = 0$

$(x^2 - 2x + 1) + (y^2 + 4y + 4) = -4 + 1 + 4$

$\div 2 \quad 1 \nearrow 2 \qquad \div 2 \quad 2 \nearrow 2$

$(x-1)^2 + (y+2)^2 = 1$

Center: $(1, -2)$ $r = \sqrt{1} = 1$

b. $x^2 + 2y^2 - x + 10y + 25 = 0$

Coefficient not same ≠ circle

No

Circle P is represented by the equation $(x-4)^2 + (y+1)^2 = 36.$

Center: $(4, -1)$ $r = 6$

a. Determine the equation of a circle that has the same center as circle P but whose circumference is twice that of circle P.

* center does not change

Circle P $C = 2\pi r$

$\rightarrow C = 12\pi$

$(x-4)^2 + (y+1)^2 = 144$

new circle $\rightarrow \frac{24\pi}{2\pi} = \frac{2\pi r}{2\pi}$ $r = 12$

b. Determine the equation of a circle that has the same center as circle P but whose circumference is three times that of circle P.

* center does not change

New circle $\rightarrow 36\pi = 2\pi r$

$r = 18$

$(x-4)^2 + (y+1)^2 = 324$

c. Determine the equation of a circle that has the same center as circle P but whose area is twice that of circle P.

* center does not change

Circle P $A = \pi r^2$

$\rightarrow A = \pi 36$

$(x-4)^2 + (y+1)^2 = 72$

new circle $\rightarrow 72\pi = \pi r^2$

$r = \sqrt{72}$

d. Determine the equation of a circle that has the same center as circle P but whose area is three times that of circle P.

* center does not change

new circle $\rightarrow 108\pi = \pi r^2$

$r = \sqrt{108}$

$(x-4)^2 + (y+1)^2 = 108$

13.3:
Determining Points on a Circle

In this problem, you will continue to explore the connection between the Pythagorean Theorem and circles.

Consider circle A with its center point located at the origin and point P (5, 0) on the circle as shown.

There are an infinite number of points located on circle A. To determine the coordinates of other points located on circle A, you can use the Pythagorean Theorem.

1. Use the Pythagorean Theorem to determine if point B (4, 3) lies on circle A, and then explain your reasoning.

AP = radius

radius = 5

$\boxed{\text{* hypotenuse is radius}}$

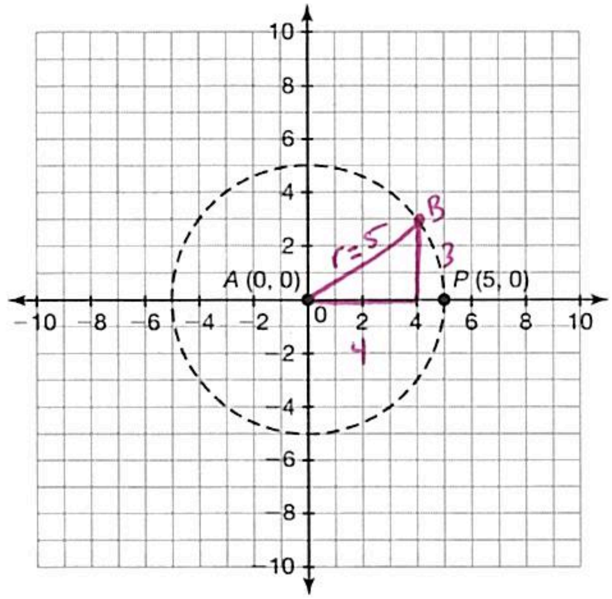
$a^2 + b^2 = c^2$

$4^2 + 3^2 = 5^2$

$16 + 9 = 25$

$25 = 25$

$\boxed{\text{Yes, it lies on the circle}}$



2. Consider circle D centered at the origin with a diameter of 16 units as shown. Use the Pythagorean Theorem to determine if point H $(5, \sqrt{38})$ lies on circle D, and then explain your reasoning.

d = 16

r = 8

$\boxed{\text{* hypotenuse is radius}}$

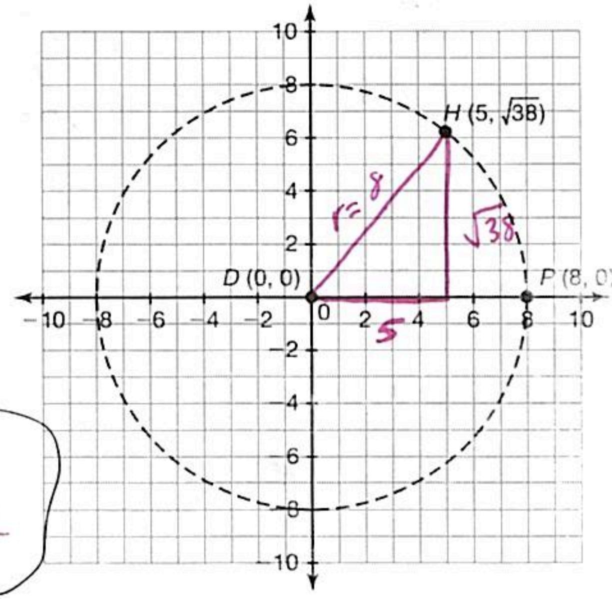
$a^2 + b^2 = c^2$

$5^2 + \sqrt{38}^2 = 8^2$

$25 + 38 = 64$

$63 = 64$

$\boxed{\text{No, it is not on the circle}}$



3. Consider circle E centered at the origin with a diameter of 34 units as shown.

a. Verify that point J (8, 15) lies on circle E. Explain your reasoning.

$d = 34$

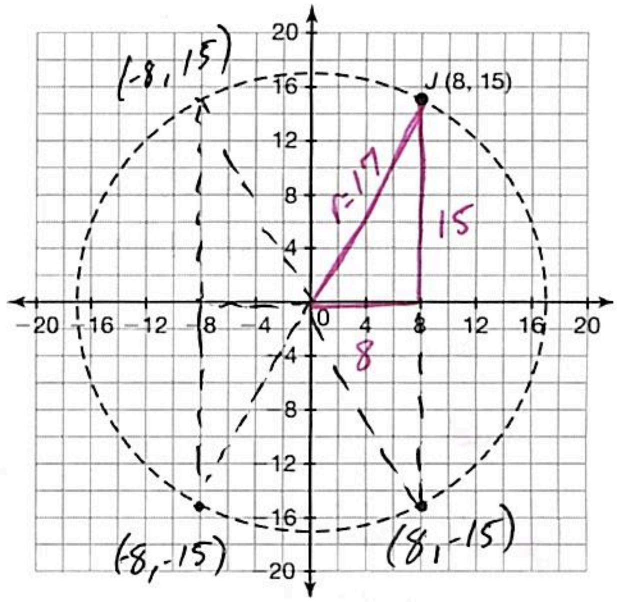
$r = 17$

$a^2 + b^2 = c^2$

$8^2 + 15^2 = 17^2$

$64 + 225 = 289$

$289 = 289$ ✓ yes



b. Use symmetry to determine 3 more points on circle E.

$(8, -15)$

$(-8, -15)$

$(-8, 15)$

4. Consider circle G with its center point located at (3, 0) and point M (3, 2) on the circle. Determine whether each point lies on circle G, and then explain your reasoning.

(c) (c)

* $r = 2$

J $(4.5, \frac{\sqrt{3}}{2})$ → $3 \to 4.5$ $\quad 0 \to \frac{\sqrt{3}}{2}$

1.5 (a) $\quad \frac{\sqrt{3}}{2}$ (b)

$1.5^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = 2^2$

$2.25 + .75 = 4$

$3 = 4$ X NO

P $(4, \sqrt{3})$

$3 \to 4 \quad 0 \to \sqrt{3}$

1 (a) $\quad \sqrt{3}$ (b)

$1^2 + \sqrt{3}^2 = 2^2$

$1 + 3 = 4$

$4 = 4$ ✓ yes