Grasshoppers Everywhere!

3.3

Area and Perimeter of Parallelograms on the Coordinate Plane

LEARNING GOALS

In this lesson, you will:

- Determine the perimeter of parallelograms on a coordinate plane.
- Determine the area of parallelograms on a coordinate plane.
- Determine and describe how proportional and non-proportional changes in the linear dimensions of a parallelogram affect its perimeter and area.
- Explore the effects that doubling the area has on the properties of a parallelogram.

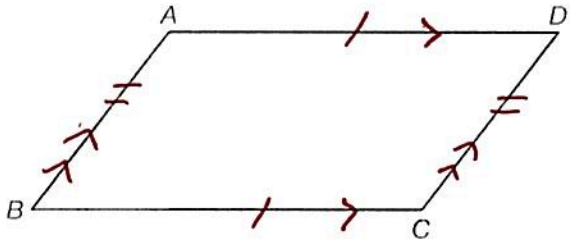
You wouldn't think that grasshoppers could be dangerous. But they can damage farmers' crops and destroy vegetation. In 2003, a huge number of grasshoppers invaded the country of Sudan, affecting nearly 1700 people with breathing problems.

Grasshopper invasions have been recorded in North America, Europe, the Middle East, Africa, Asia, and Australia. One of the largest swarms of grasshoppers—known as a "cloud" of grasshoppers—covered almost 200,000 square miles!

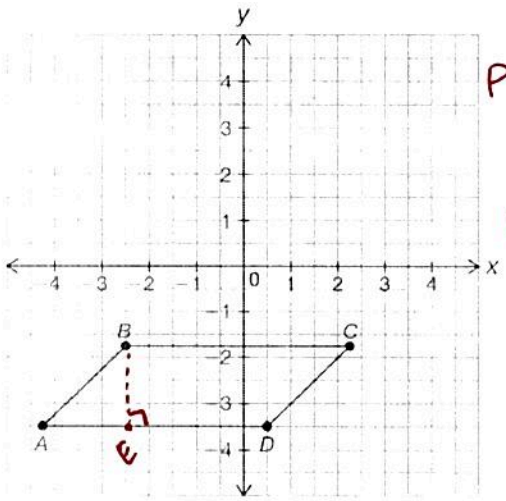
PROBLEM 1 Rectangle or . . . ?

You know the formula for the area of a parallelogram. The formula $A = bh$, where A represents the area, b represents the length of the base, and h represents the height, is the same formula that is used when determining the area of a rectangle. But how can that be if they are different shapes?

1. Use the given parallelogram to explain how the formula for the area of a parallelogram and the area of a rectangle can be the same.



2. Analyze parallelogram *ABCD* on the coordinate plane.



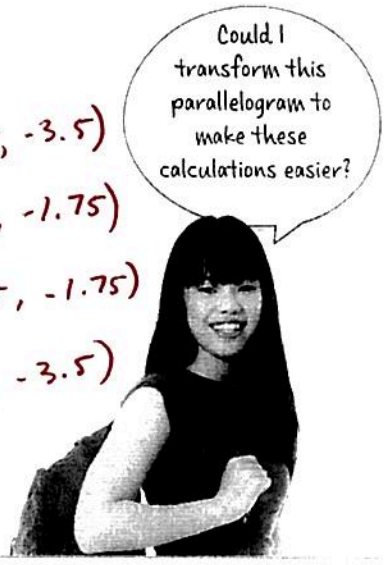
Points:

A (-4.25, -3.5)

B (-2.5, -1.75)

C (2.25, -1.75)

D (0.5, -3.5)



a. Determine the perimeter of parallelogram *ABCD*.

$$AB = \sqrt{(-2.5-(-4.25))^2 + (-1.75-(-3.5))^2}$$
$$= \sqrt{1.75^2 + 1.75^2}$$
$$= \sqrt{6.125}$$
$$= 2.47$$

$$BC = 2.25 - (-2.5)$$
$$= 4.75$$

Perimeter
$2.47(2) + 4.75(2)$
$= 14.44$

* Don't need to find BCD and AD because

$AB \cong CD$ and $BC \cong AD$

b. To determine the area of parallelogram *ABCD*, you must first determine the height. Describe how to determine the height of parallelogram *ABCD*.

Find length of perpendicular line segment from base to vertex opposite the base.

c. Ms. Finch asks her class to identify the height of parallelogram *ABCD*. Pete draws a perpendicular line from point *D* to *AD*, saying that the height is represented by $\overline{BE}$. Tonya disagrees. She draws a perpendicular line from point *D* to *BC*, saying that the height is represented by *DF*. Who is correct? Explain your reasoning.

d. Determine the height of parallelogram *ABCD*.

Change in y-value

E(-2.5, -3.5) EB = -1.75 - (-3.5)

= 1.75

e. Determine the area of parallelogram *ABCD*.

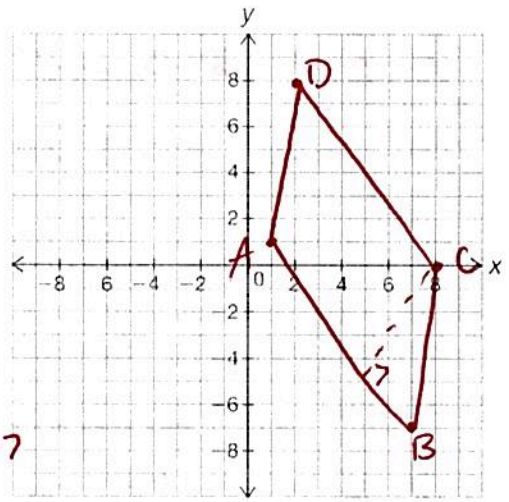
A = bh

A = (4.75)(1.75)

A = 8.31

PROBLEM 2 Stand Up Straight!

1. Graph parallelogram *ABCD* with vertices *A* (1, 1), *B* (7, −7), *C* (8, 0), and *D* (2, 8). Determine the perimeter.



* To find perimeter

- find side measure of AD and AB

- AD ≅ CB AB = 10

AB ≅ DC AD = 7.07

P = 34.14

* Area on pg. 293

2. Determine the area of parallelogram *ABCD*.

a. Using *CD* as the base, how will determining the height of this parallelogram be different from determining the height of the parallelogram in Problem 1?

b. Using *CD* as the base, explain how you will locate the coordinates of point *E*, the point where the base and height intersect.

Area

c. Determine the ~~coordinates of point *E*.~~ Label point *E* on the coordinate plane.

① m_{AB} (base) = 10

② Slope AB (base) = $-\frac{4}{3}$

③ ⊥ Slope (height) = $\frac{3}{4}$

④ equation for height

$y - y_1 = m(x - x_1)$ * use point "C"

$y - 0 = \frac{3}{4}(x - 8)$

$y = \frac{3}{4}x - \frac{24}{4}$

$y = \frac{3}{4}x - 6$

⑤ find intersection point (system of equations)

$\frac{3}{4}x - 6 = -\frac{4}{3}x + \frac{7}{3}$

(common denomenator)

$\frac{9}{12}x - \frac{72}{12} = \frac{-16}{12}x + \frac{28}{12}$

$\frac{25}{12}x = \frac{100}{12}$

$25x = 100$

$x = 4$

$y = \frac{3}{4}(4) - 6$

$y = \frac{12}{4} - 6$

$y = -3$

⑥ distance of height (use "C" and int. point)

$d = \sqrt{(0+3)^2 + (8-4)^2}$

$d = 5$

4.5 equation for base

$y - y_1 = m(x - x_1)$

$y - 1 = -\frac{4}{3}(x - 1)$

$y - 1 = -\frac{4}{3}x + \frac{4}{3}$

$y = -\frac{4}{3}x + \frac{7}{3}$

⑦ $A = bh$

$A = (10)(5) = 50$ ★

d. Determine the height of parallelogram *ABCD*.

e. Determine the area of parallelogram *ABCD*.

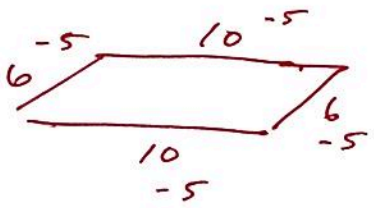
3

3. You determined earlier that any side of a parallelogram can be thought of as the base. Predict whether using a different side as the base will result in a different area of the parallelogram. Explain your reasoning.

PROBLEM 5 Reaching for New Heights, and Bases . . . One More Time

Recall that you have determined and described how proportional and non-proportional changes in the linear dimensions of rectangles and triangles affect their perimeter and area.

1. Determine and describe how subtracting 5 units from all sides of a parallelogram will affect the perimeter of the resulting parallelogram. Provide an example and explain your reasoning.



dimensions → perimeter

$-5\ (4) \longrightarrow -20$

$-10\ (4) \longrightarrow -40$

$-100\ (4) \longrightarrow -400$

Non-proportional

(+/-)

2. Describe how multiplying the base and height of a parallelogram by a factor of 10 will affect the area of the resulting parallelogram. Provide an example, determine its area, and explain your reasoning.

dimensions → perimeter → area

$\times 10 \longrightarrow \times 10 \longrightarrow \times 100$

$\times 5 \longrightarrow \times 5 \longrightarrow \times 25$

$\times 3 \longrightarrow \times 3 \longrightarrow \times 9$

$\times \frac{1}{2} \longrightarrow \times \frac{1}{2} \longrightarrow \times \frac{1}{4}$

Proportional

(×/÷)