PAP Geometry

Coach C's Notes

10.4 - 10.5 Notes
Interior and Exterior Angles of Polygons

Chapter 10 Vocab:

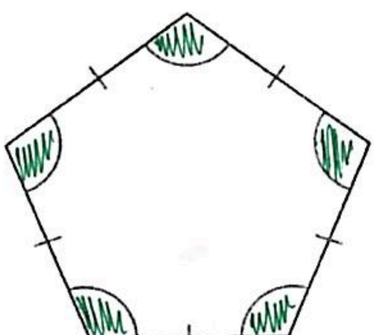
A convex polygon is defined as a polygon with each of its *interior angles* less than 180°. This means that all the vertices of the polygon will point outwards.

A concave polygon is a polygon with one or more *interior angles* greater than 180°. It looks like a vertex has been 'pushed in' towards the inside of the polygon. ** Think- "it *caves* inwards" **

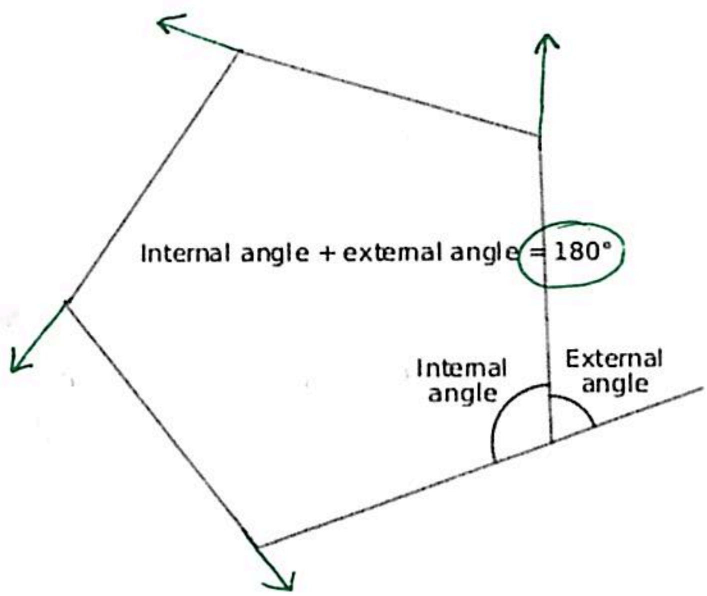
Interior Angle of a polygon is an angle which faces the inside of the polygon and is formed by consecutive sides of a polygon.

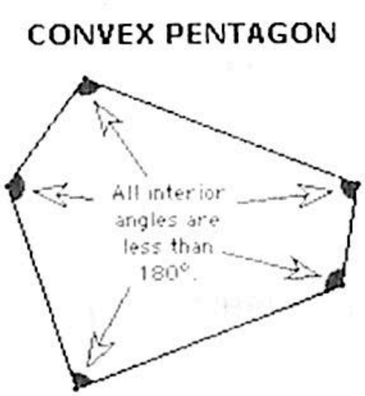
Exterior Angle of a polygon is formed adjacent to each interior angle by extending one side of each vertex of the polygon.

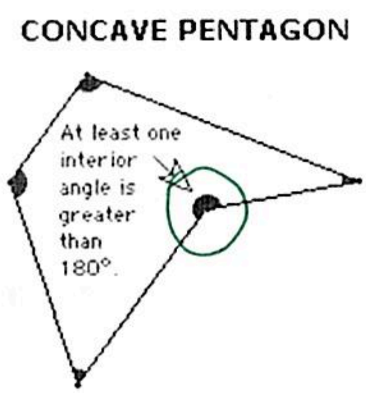
Regular polygon: a polygon is regular when all angles are congruent and all sides are congruent. (equilateral and equiangular)



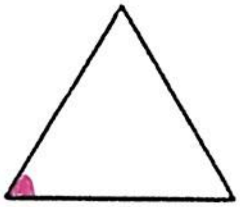
Regular pentagon



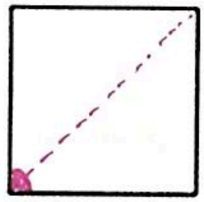




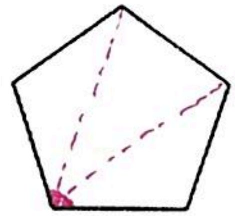
Naming Polygons: Polygons are named by using the Greek prefix of their number of sides and the suffix -*gon*.



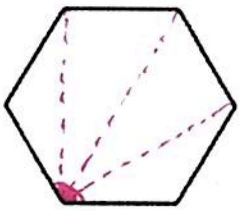
Triangle



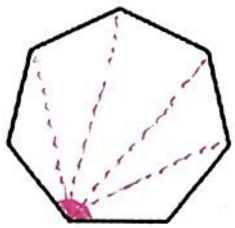
Quadrilateral



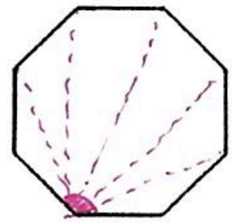
Pentagon



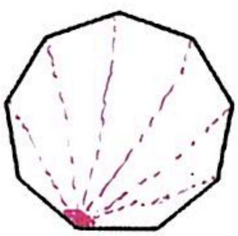
Hexagon



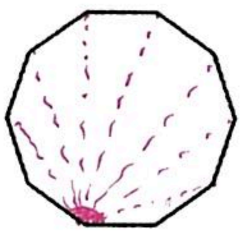
Heptagon



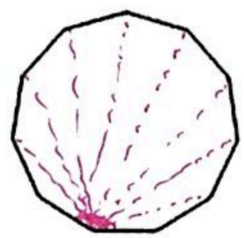
Octagon



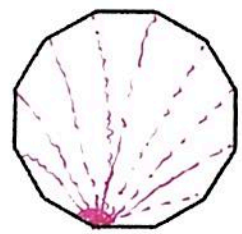
Nonagon



Decagon



Undecagon/ Hendecagon



Dodecagon

Choose one vertex in each polygon and draw all possible diagonals from that vertex.

Number of Sides of Polygon	3	4	5	6	7	8	9	10	11	12
Number of Diagonals Drawn	0	1	2	3	4	5	6	7	8	9
Number of Triangles Formed	1	2	3	4	5	6	7	8	9	10
Sum of the Measure of Interior Angles	180°	360°	540°	720°	900°	1080°	1260°	1440°	1620°	1800°

Interior Angles:

We can use the table to derive the formula for the sum of interior angles of an *n*-sided polygon:

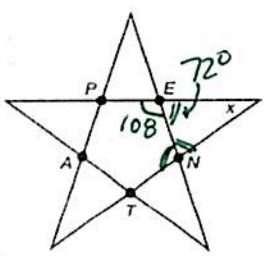
$$\boxed{180°(n-2)}$$

**multiply the # of triangles formed by 180°

If a regular polygon has *n* sides, you can find the measure of each individual angle using:

$$\boxed{\frac{180°(n-2)}{n}}$$

1. *PENTA* is a regular pentagon. Solve for *x*.



$$\frac{180(n-2)}{n}$$

$$\frac{180(5-2)}{5} = 108°$$

$180 - 108 = 72$

$72 + 72 + x = 180$

$144 + x = 180$

$\boxed{x = 36°}$

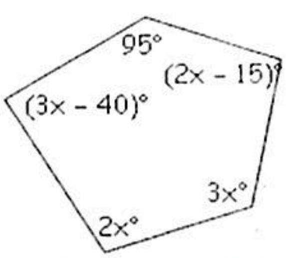
2. The Susan B. Anthony dollar coin minted in 1999 features a regular 11-gon, or hendecagon, inside a circle on both sides of the coin.

What is the measure of each interior angle of the regular hendecagon?

$$\frac{180(n-2)}{n} \rightarrow \frac{180(11-2)}{11}$$

$$\frac{180(9)}{11} = \frac{1620}{11} = \boxed{147.27°}$$

3. Find the value of x.



$180(5-2) = 180(3) = \underline{540°}$

$540° = 95 + 3x - 40 + 2x - 15 + 2x + 3x$

$540 = 40 + 10x$

$-40 \quad -40$

$500 = 10x$

$\boxed{x = 50}$

4-a. What is the sum of the interior angles of a polygon with 22 sides?

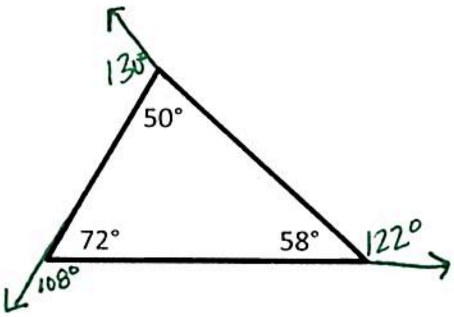
$180(n-2) \rightarrow 180(22-2) = 180(20)$

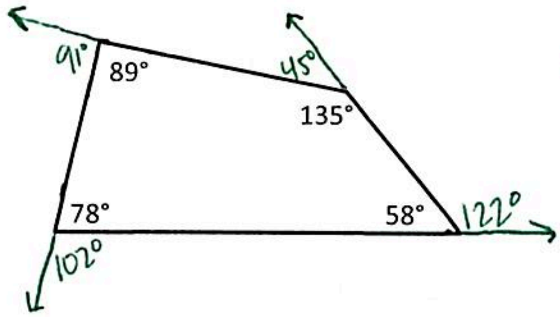
$\boxed{3600°}$

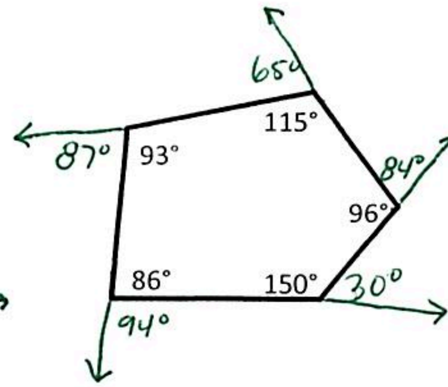
4-b. What would each interior angle measure if the polygon was regular?

$$\frac{3600}{n} = \frac{3600}{22} = \boxed{163.64°}$$

Exterior Angles: each interior angle of a polygon can be paired with an interior angle







Number of Sides of the Polygon	3	4	5	6	10	20
Number of Linear Pairs Formed	3	4	5	6	10	20
Sum of Measures of Linear Pairs	540°	720°	900°	1080°	1800°	3600°
Sum of Measures of Interior Angles	180°	360°	540°	720°	1440°	3240°
Sum of Measures of Exterior Angles	360°	360°	360°	360°	360°	360°

**The sum of the exterior angle measures of all polygons is always equal to 360° **

The measure of each exterior angle of an *n*-sided ***regular*** polygon can be found using:

$$\boxed{\frac{360°}{n}}$$

1. If the measure of each exterior angle of a regular polygon is 18°, how many sides does it have?

$(\cancel{n})\frac{360}{\cancel{n}} = 18(n)$

$\frac{360}{18} = \frac{\cancel{18}(n)}{18}$

$\boxed{n = 20 \text{ sides}}$

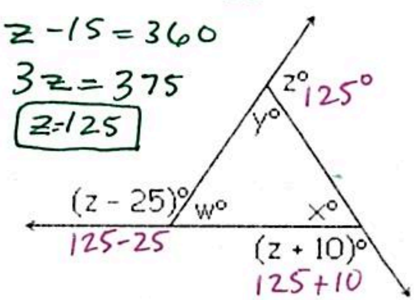
2. Find the value of each variable.

$z - 25 + z + z + 10 = 360$

$3z - 15 = 360$

$3z = 375$

$\boxed{z = 125}$



$180 - 125 = y$

$\boxed{y = 55°}$

$180 - (125 + 10) = x$

$\boxed{x = 45}$

$180 - (125 - 25) = w$

$\boxed{w = 80°}$