

## Angle Postulates and Theorems

### LEARNING GOALS

In this lesson, you will:

- Use the Corresponding Angle Postulate.
- Prove the Alternate Interior Angle Theorem.
- Prove the Alternate Exterior Angle Theorem.
- Prove the Same-Side Interior Angle Theorem.
- Prove the Same-Side Exterior Angle Theorem.

### KEY TERMS

- Corresponding Angle Postulate
- conjecture
- Alternate Interior Angle Theorem
- Alternate Exterior Angle Theorem
- Same-Side Interior Angle Theorem
- Same-Side Exterior Angle Theorem

You are constantly bombarded with information through magazines, newspapers, television, and the Internet. However, not all "facts" that you read about are actually true! If you want to be an educated consumer of information, you should always be looking for the argument, or proof, to back up a statement. If you can't find such information then you should be skeptical.

Sometimes you need to carefully examine the evidence. For example, say someone claims that 4 out of 5 dentists recommend a certain toothpaste. Sounds pretty impressive, right? However, what if you learned that only five dentists were asked their opinions? You might start to question the claim. What if you also learned that the dentists were paid by the toothpaste company for their opinions? As you can see, sometimes the "truth" isn't always what it appears to be.

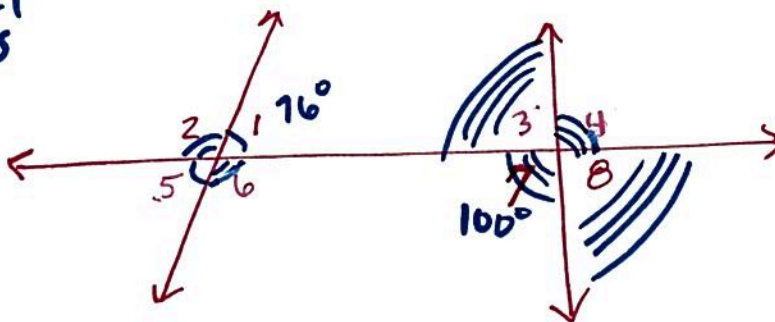
# PROBLEM 1 Investigating Patterns of Geometric Relationships



1. Draw a transversal intersecting two non-parallel lines, and number each angle. Then use a protractor to determine each angle measure.

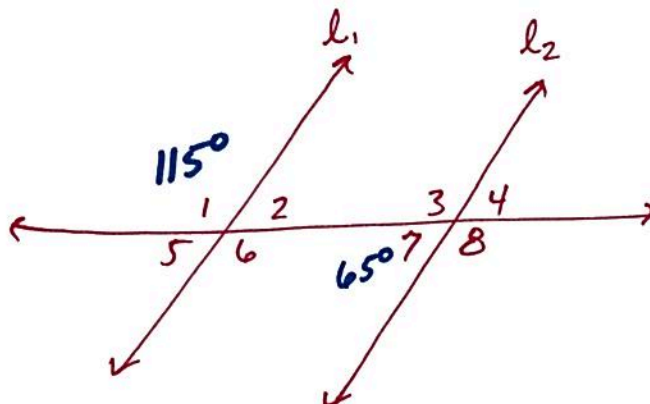
Given:  $m\angle 7 = 100^\circ$   
 $m\angle 1 = 76^\circ$

\*Vertical angles are 112



Find:  
 $m\angle 2 = 104^\circ$   
 $m\angle 3 = 80^\circ$   
 $m\angle 4 = 100^\circ$   
 $m\angle 5 = 76^\circ$   
 $m\angle 6 = 104^\circ$   
 $m\angle 8 = 80^\circ$

2. Draw a transversal intersecting two parallel lines, and number each angle. Then use a protractor to determine each angle measure.



Given:  $l_1 \parallel l_2$   
 $m\angle 1 = 115^\circ$   
 $m\angle 7 = 65^\circ$

Find:  
 $m\angle 2 = 65^\circ$   
 $m\angle 3 = 115^\circ$   
 $m\angle 4 = 65^\circ$   
 $m\angle 5 = 65^\circ$   
 $m\angle 6 = 115^\circ$   
 $m\angle 8 = 115^\circ$

Use the information from Questions 1 and 2 to answer Questions 3 through 8.

3. What do you notice about the measures of each pair of alternate interior angles when the lines are:

a. non-parallel?

*not equal / not congruent*

b. parallel?

*have equal measure / congruent*

4. What do you notice about the measures of each pair of alternate exterior angles when the lines are:

( $\angle 5$  and  $\angle 4$ )

a. non-parallel?

not equal measures

b. parallel?

equal measures

5. What do you notice about the measures of each pair of corresponding angles when the lines are:

( $\angle 3$  and  $\angle 1$ )

a. non-parallel?

not equal measures

b. parallel?

equal measures

6. What do you notice about the measures of the same-side interior angles when the lines are:

( $\angle 6$  and  $\angle 7$ )

a. non-parallel?

not supplementary

b. parallel?

supplementary

7. What do you notice about the measures of the same-side exterior angles when the lines are:

( $\angle 5$  and  $\angle 8$ )

a. non-parallel?

not supplementary

b. parallel?

supplementary



8. Summarize your conclusions in the table by writing the relationships of the measures of the angles. The relationships are either congruent or not congruent, supplementary or not supplementary.

Angles	Two Parallel Lines Intersected by a Transversal	Two Non-Parallel Lines Intersected by a Transversal
Alternate Interior Angles	$\cong$	not $\cong$
Alternate Exterior Angles	$\cong$	not $\cong$
Corresponding Angles	$\cong$	not $\cong$
Same-Side Interior Angles	Supplementary	not supp.
Same-Side Exterior Angles	Supplementary	not supp.

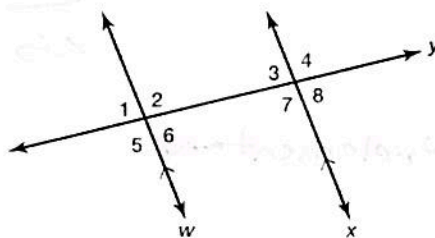


9. Use your table in Question 8 to compare your conclusions with other groups or classmates. Investigate the patterns to make a conjecture about the geometric relationships. Also, compare the measures of the angles everyone used.

## PROBLEM 2 The Corresponding Angle Postulate



The **Corresponding Angle Postulate** states: "If two parallel lines are intersected by a transversal, then corresponding angles are congruent."



1. Name all pairs of angles that are congruent using the Corresponding Angle Postulate.

$$\begin{aligned} \angle 1 &\cong \angle 3 & \angle 5 &\cong \angle 7 \\ \angle 2 &\cong \angle 4 & \angle 6 &\cong \angle 8 \end{aligned}$$

A **conjecture** is a hypothesis that something is true. The hypothesis can later be proved or disproved.

2. Write a conjecture about each pair of angles formed by parallel lines cut by a transversal. Explain how you made each conjecture.

- a. alternate interior angles.

If two parallel lines are intersected by a transversal, then alt. interior  $\angle$ 's are  $\cong$ .

$\angle 2$  and  $\angle 7 \rightarrow$  alt. int.  $\angle$ 's

$\angle 2 \cong \angle 4 \rightarrow$  Corresponding  $\angle$  post.

$\angle 4 \cong \angle 7 \rightarrow$  vert.  $\angle$  Theorem

\*  $\angle 2 \cong \angle 7 \rightarrow$  Transitive

- b. alternate exterior angles.

If two  $\parallel$  lines are intersected by a transversal, then alt. exterior  $\angle$ 's are  $\cong$ .

$\angle 4$  and  $\angle 5 \rightarrow$  alt. ext.  $\angle$ 's

$\angle 4 \cong \angle 2 \rightarrow$  corresponding  $\angle$  post.

$\angle 2 \cong \angle 5 \rightarrow$  vert.  $\angle$  Theorem

\*  $\angle 4 \cong \angle 5 \rightarrow$  Transitive

- c. same-side interior angles

If two  $\parallel$  lines are intersected by a transversal, then same-side interior  $\angle$ 's are supplementary.

$\angle 2$  and  $\angle 3 \rightarrow$  S.S. int.  $\angle$ 's

$\angle 2 \cong \angle 4 \rightarrow$  Corr.  $\angle$  post.

$\angle 3$  and  $\angle 4 \rightarrow$  supp.

$\angle 2$  and  $\angle 3$  supp.  $\rightarrow$  substitution

Test it out! Conjecture is all about testing.



d. same-side exterior angles

IF two  $\parallel$  lines are intersected by a transversal, then same-side exterior  $\angle$ 's are supplementary.

$\angle 1$  and  $\angle 4 \rightarrow$  S.S. ext.  $\angle$ 's

$\angle 1 \cong \angle 3 \rightarrow$  corr.  $\angle$  post.

$\angle 3$  and  $\angle 4 \rightarrow$  supp.

$\angle 1$  and  $\angle 4$  supp.  $\rightarrow$  substitution



3. Did you use inductive or deductive reasoning to make each conjecture?

Inductive

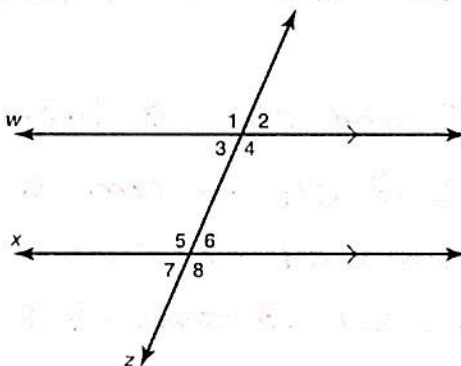
### PROBLEM 3 Conjecture or Theorem?



1. Consider the statement about alternate interior angles, "If two parallel lines are intersected by a transversal, then alternate interior angles are congruent."
- a. Determine whether the statement is an undefined term, a definition, a postulate, a conjecture, or a theorem. Explain your reasoning.

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- b. Use the diagram to write the "Given" and "Prove" statements for the alternate interior angle statement in Question 1.

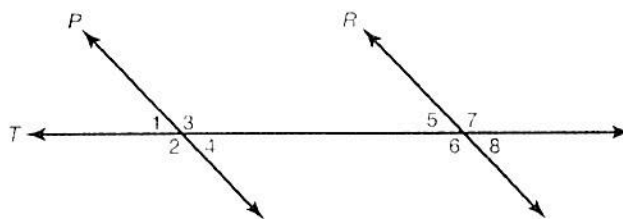


Given:

Prove:

## Read Between the Lines

1. Determine each angle measure, if possible.



Given:  $m\angle 4 = 37^\circ$

- Identify the angle that forms a vertical angle pair with  $\angle 4$  and determine its measure. Explain your reasoning.
  - Identify the two angles that form a linear pair with  $\angle 4$  and determine their measures. Explain your reasoning.
2. Gail determined the measures of all eight angles labeled using the given information. Stu said she could only calculate the measure of four angles with certainty. Who is correct? Explain your reasoning.



