 Name: Coach C's Notes

Chapter 14 – 15 : Probability

(Day 1)

Fundamental Counting Principal:

If there are n items and m_1 ways to choose a first item, m_2 ways to choose a second item after the first item has been chosen, and so on, then there are $m_1 * m_2 * \ldots * m_n$ ways to choose n items.

Example 1: To make a yogurt parfait, you choose one flavor of yogurt, one fruit topping, and one additional topping. How many parfait choices are there?

Flavor	**Fruit**	**Toppings**
Plain	Peaches	Peanuts
Vanilla	Strawberries	Oreo
Chocolate	Bananas	Graham Cracker
Strawberry	Raspberries	Whipped Cream
	Blueberries	Almonds
		Candy

(4) x (5) x (6) = $\boxed{\text{120 Choices}}$

Example 2: A password for a site consists of 4 digits followed by 2 letters. Each letter and digit can be used more than once. How many unique passwords are possible?

digits = 0-9 (10)
letters = A-Z (26)

$\frac{D}{(10)} \times \frac{D}{(10)} \times \frac{D}{(10)} \times \frac{D}{(10)} \times \frac{L}{(26)} \times \frac{L}{(26)} = \boxed{6{,}760{,}000}$

Example 3: You need to create a password with 5 letters. Letters can be used more than once and can be uppercase or lowercase which are considered different. How many passwords are possible?

Uppercase (26)
lowercase (26)
52

$\frac{L}{(52)} \times \frac{L}{(52)} \times \frac{L}{(52)} \times \frac{L}{(52)} \times \frac{L}{(52)} = \boxed{380{,}204{,}032}$

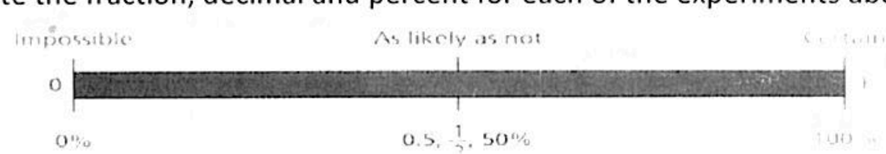
Intro to Probability:

<u>Probability</u> is the measure of how likely an event is to occur. Each possible result of a probability experiment or situation is the **<u>outcome</u>**. The **<u>sample space</u>** is the set of *<u>all</u>* possible outcomes. An **<u>event</u>** is an outcome or a set of outcomes.

Experiment or Situation	Rolling a 6-sided die	Spinning a 4-color spinner	Flipping a coin
Sample Space	{1, 2, 3, 4, 5, 6}	{blue, yellow, green, red}	{heads, tails}
	$\frac{1}{6}$ or 0.1$\overline{6}$ or 16.7%	$\frac{1}{4}$ or 0.25 or 25%	$\frac{1}{2}$ or 0.5 or 50%

Probabilities are written as fractions or decimals from 0 to 1, or as a percent from 0% to 100%

write the fraction, decimal and percent for each of the experiments above…



Equally likely outcomes have the same chance of occurring. When you toss a fair coin, heads and tails are equally likely outcomes. **Favorable outcomes** are outcomes in a specified event. For equally likely outcomes, the **theoretical probability** of an event is the ratio of the number of favorable outcomes to the total number of outcomes.

Theoretical Probability:

$$\boxed{P(event) = \frac{number\ of\ favorable\ outcomes}{total\ number\ of\ outcomes\ (sample\ space)}}$$

Example 1: Each letter of the word PROBABLE is written on a separate card. The cards are placed face down and mixed up. What is the probability that a randomly selected card has a consonant?

P(drawing a consonant) = $\frac{5 \text{ favorable outcomes}}{8 \text{ total outcomes}} = \frac{5}{8}$ or 62.5 %

Example 2: Two dice are rolled, one red/one blue. What is the probability that the numbers have a sum of 4?

(R/B) total: 6 x 6 = 36

favorable: 1/3

3/1

2/2

P(sum 4) = $\frac{3}{36} = \frac{1}{12}$ or 8.3%

Example 3: Two dice are rolled, one red/one blue. What is the probability that the red die is greater?

favorable: (R/B) 5/1 5/2 6/3 total: 36

2/1 6/1 6/2 5/4

3/1 3/2 4/3 6/4

4/1 4/2 5/3 6/5

P(red greater) = $\frac{15}{36} = \frac{5}{12}$ or 41.7

The sum of all probabilities in the sample space is 1. The **complement** of an event E is the set of all outcomes in the sample space that are not in E.

Complement:

The probability of the complement of event E is

$$P(not\ E) = 1 - P(E)$$

Example 1: There are 25 kids in activity period. The table shows the students who are studying a foreign language. What is the probability that a randomly selected student is not studying a foreign language?

Language	Number of Students
French	6
Spanish	12
Japanese	3

P(not studying) = 1 − P(studying)

P(not studying) = $1 - \frac{21}{25}$

$= \frac{4}{25}$ or 16 %

Example 2: Two integers from 1 to 10 are randomly selected, the same number may be chosen twice. What is the probability that both numbers are less than 9?

$P(\# < 9) = 1 - P(\# \geq 9)$

$P(\# < 9) = 1 - \frac{2}{10}$

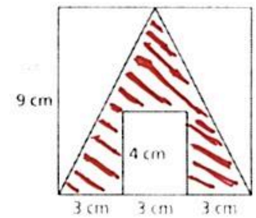
$= 8/10$

Both #'s less than 9

$\frac{8}{10} \times \frac{8}{10} = \frac{64}{100} = \frac{16}{25}$ or 64 %

Geometric Probability is a form of theoretical probability determined by a ratio of lengths, areas, or volumes.

Example 1: A figure is created placing a rectangle inside a triangle inside a square as shown. If a point inside the figure is chosen at random, what is the probability that the point is inside the shaded region?



Area

Square = 81 cm^2

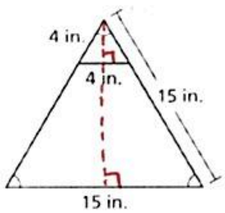
Triangle = 40.5 cm^2

Rectangle = 12 cm^2

$P(\text{inside shaded}) = \frac{40.5 - 12}{81} = \frac{28.5}{81}$

$= 35.2\%$

Example 2: Find the probability that a point chosen at random is inside the small triangle?



Area

large = $\frac{15\sqrt{168.75}}{2}$

Small = $\frac{4\sqrt{12}}{2} = 2\sqrt{12}$

$P(\text{small } \triangle) = \frac{2\sqrt{12}}{7.5\sqrt{168.75}} = \frac{6.93}{97.43} = 7.1\%$

You can estimate the probability of an event by using data, or by **experiment**. For example: if a doctor states that an operation "has an 80% probability of success", that is an estimate based on similar case histories.

Each repetition of an experiment is a **trial**. The sample space of an experiment is the set of all possible outcomes. The **experimental probability** of an event is the ratio of the number of times that the event occurs (*the frequency*) to the number of trials.

Experimental Probability:

$$\boxed{\textit{experimental probability} = \frac{\textit{number of times the event occurs}}{\textit{number of trials}}}$$

Example 1: The table shows the results of a spinner experiment.

Number	Occurrences
1	6
2	11
3	19
4	14

total: 50

Find the experimental probability of:

a. Spinning a 4 $P = \frac{14}{50} = \frac{7}{25}$ or 28%

b. Spinning a number greater than 2

$P = \frac{19+14}{50} = \frac{33}{50}$ or 66%

Example 2: The table shows the results of a dice experiment.

Number	Occurrences
1	15
2	17
3	10
4	16
5	23
6	19

total: 100

Find the experimental probability of:

a. Rolling an even number

$P = \frac{17+16+19}{100} = \frac{52}{100} = \frac{13}{25}$ or 52%

b. Rolling a number less than 5

$P = \frac{15+17+10+16}{100} = \frac{58}{100} = \frac{29}{50}$ or 58%

END DAY 1

(GROUP EXIT TICKET

Chapter 14 – 15 : Compound Events

(Day 2)

A **simple event** is an event that describes a single outcome. A **compound event** is an event made up of two or more simple events. **Mutually exclusive events** are events that cannot both occur in the same trial of an experiment. For example, rolling a 1 and rolling a 2 on the same roll of a die are mutually exclusive events.

* Mutually Exclusive Events:

For two mutually exclusive events A and B,

$$\boxed{P(A \cup B) = P(A) + P(B)}$$

Remember!
Recall that the union symbol $\cup$ means "or."

$\cup$ = "or"

Example 1: When a die is rolled, $P(less\ than\ 3) =\ ?$

$$P(1 \text{ or } 2) = P(1) + P(2)$$
$$= \frac{1}{6} + \frac{1}{6} = \boxed{\frac{1}{3}}$$

Example 2: A group of students is donating blood during a blood drive. A student has a $\frac{9}{20}$ probability of having type O blood and a $\frac{2}{5}$ probability of having type A blood.

a. Explain why the events "type O" and "type A" are mutually exclusive.

You can only have one type of blood

b. What is the probability that a student has type O or type A blood?

$$P(O \text{ or } A) = P(O) + P(A)$$
$$= \frac{9}{20} + \frac{2}{5}\left(\frac{4}{4}\right) \rightarrow \frac{9}{20} + \frac{8}{20} = \boxed{\frac{17}{20}}$$

Example 3: Each student cast one vote for Prom Queen. Of the students, 25% votes for Heather, 20% for Claire, and 55% for Vanessa. What is the probability that a student chosen at random voted for Claire or Vanessa?

$$P(\text{Claire or Vanessa}) = P(C) + P(V)$$
$$= 20\% + 55\% = \boxed{75\%}$$

Inclusive events are events that have one or more outcomes in common. For example, when you roll a die "rolling an even number" and "rolling a prime number" are not mutually exclusive. The number 2 is both even and prime, so the events are inclusive.

* Inclusive Events:

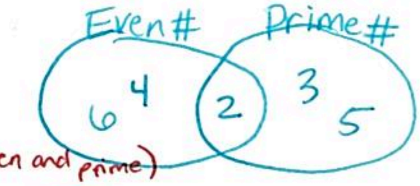
For two inclusive events A and B,

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

$\cap$ = "and"

Remember!
Recall that the intersection symbol $\cap$ means "and."

Example 1: When a die is rolled, $P(\text{even or prime number}) = \ ?$



$P(\text{even or prime}) = P(\text{even}) + P(\text{odd}) - P(\text{even and prime})$

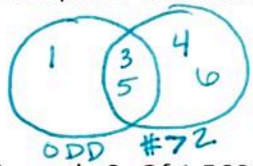
$= \frac{3}{6} + \frac{3}{6} - \frac{1}{6} = \boxed{\frac{5}{6}}$

* talk about complemen[t]

$\frac{1}{6}$ of getting a 1

which is the only non-even or odd

Example 2: Find the probability of rolling an odd number or a number greater than 2.



$P(\text{odd or} > 2) = P(\text{odd}) + P(>2) - P(\text{odd and} > 2)$

$= \frac{3}{6} + \frac{4}{6} - \frac{2}{6} = \boxed{\frac{5}{6}}$

Example 3: Of 1,560 students surveyed, 840 were seniors and 630 watched the news daily. The rest of students were juniors and only 215 of the news watchers were juniors. What is the probability that a student was a senior or watched the news daily?

$630 - 215 = 415$ seniors/news

$P(\text{senior or news}) = P(\text{senior}) + P(\text{news}) - P(\text{senior and news})$

$= \frac{840}{1560} + \frac{630}{1560} - \frac{415}{1560} = \frac{1055}{1560} = \boxed{\frac{211}{312} \text{ or } 67.6\%}$

Events are **<u>independent events</u>** if the occurrence of one event *does not affect* the probability of the other. For example, when a coin is tossed twice, the outcome of the first toss landing heads up does not affect the probability of the second toss landing heads up again.

To find the probability of landing heads up twice, multiply the individual probabilities, $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

* Independent Events:

If A and B are independent events, then

$$P(A \text{ and } B) = P(A) \times P(B)$$

Example 1: A six-sided cube is labeled with the numbers 1, 2, 2, 3, 3, and 3. Four sides are colored red, one white, and one yellow.

Find the probability of:

a. Rolling a 2, then a 2.

$P(2 \text{ and } 2) = \frac{2}{6} \times \frac{2}{6} = \frac{4}{36} = \boxed{\frac{1}{9} \text{ or } 11.1\%}$

b. Rolling red, then white, then red.

$P(\text{red and white and red}) = \frac{4}{6} \times \frac{1}{6} \times \frac{4}{6} = \frac{16}{216} = \boxed{\frac{2}{27} \text{ or } 7.4\%}$

c. Rolling a 3 then rolling red.

$P(3 \text{ and red}) = \frac{3}{6} \times \frac{4}{6} = \frac{12}{36} = \boxed{\frac{1}{3} \text{ or } 33.3\%}$

Example 2: What is the probability of tossing a coin 3 times and landing on heads twice then tails?

$P(H, H, T) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \boxed{\frac{1}{8} \text{ or } 12.5\%}$

Events are **dependent events** if the occurrence of one event *affects* the probability of the other. For example, if you have 3 lemons and 1 lime in a bag and you pull out two pieces of fruit, the probabilities change depending on the outcome of the first fruit taken out.

The probability of a specific event can be found by multiplying the probabilities that make up the event.

The probability of drawing two lemons is $\frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$.

$\boxed{4}$ Lemon $\frac{3}{4}$ < 2/3, 1/3
Lime $\frac{1}{4}$ < 3/3, 0

To find the probability of dependent events, you can use conditional probability $P(B|A)$, the probability of event B, given that event A has occurred.

* Dependent Events:

If A and B are dependent events, then

$$P(A \text{ and } B) = P(A) \times P(B|A)$$

Example 1: A bag contains 10 marbles – 2 blue, 3 white, and 5 red. If marbles are selected at random, find the probability of:

a. Selecting a white marble, replacing it, and then selecting a red.

$\frac{3}{10} \times \frac{5}{10} = \frac{15}{100} = \boxed{\frac{3}{20} \text{ or } 15\%}$

b. Selecting a white and then a red, without replacing the first.

$\frac{3}{10} \times \frac{5}{9} = \frac{15}{90} = \boxed{\frac{1}{6} \text{ or } 16.7\%}$

c. Selecting 3 non-red marbles without replacement

$\frac{5}{10} \times \frac{4}{9} \times \frac{3}{8} = \frac{60}{720} = \boxed{\frac{1}{12} \text{ or } 8.3\%}$

Conditional probability often applies when data fall into categories. ***Two-way tables*** *organize data in categories that run across the table in rows and down the table in columns.*

Conditional probability $P(B|A)$ → Probability of **B**, given A has occured

Example 2: The table shows domestic migration from 1995 to 2000.

If a person is randomly selected, find the probability:

Domestic Migration by Region (thousands)

Region	Immigrant	Emigrants	
	1537	2808	= 4,345
Midwest	2410	2951	= 5,361
South	5042	3243	= 8,285
West	2666	2654	= 5,320
	11,655	11,656	= 23,311

a. That an emigrant is from the West

$P(\text{west}|\text{emmigrant}) = \frac{\text{west emmigrants}}{\text{total emmigrants}} = \frac{2654}{11,656} \boxed{= 22.8\%}$

b. That someone selected from the South region is an immigrant

$P(\text{immigrant}|\text{South}) = \frac{\text{south immigrants}}{\text{total south}} = \frac{5042}{8,285} = \boxed{60.9\%}$

c. That someone selected is an emigrant and is from the Midwest $P(\text{midwest}|\text{emmigrant}) = \frac{2951}{11656}$

$P(\text{emigrant and midwest}|\text{emigrant}) = \frac{\cancel{11,656}}{23,311} \times \frac{2951}{\cancel{11,656}} = \boxed{12.7\%}$