Parallel Line Converse Theorems

LEARNING GOALS

In this lesson, you will:

- Write parallel line converse conjectures.
- Prove parallel line converse conjectures.

KEY TERMS

- converse
- Corresponding Angle Converse Postulate
- Alternate Interior Angle Converse Theorem
- Alternate Exterior Angle Converse Theorem
- Same-Side Interior Angle Converse Theorem
- Same-Side Exterior Angle Converse Theorem

Lewis Carroll is best known as the author of *Alice's Adventures in Wonderland* and its sequel *Through the Looking Glass*. However, Carroll also wrote several mathematics books, many of which focus on logic. In fact, Carroll included logic in many of his fiction books. Sometimes these took the form of "logical nonsense" such as the tea party scene with the Mad Hatter.

At one point of the scene, Alice proclaims that she says what she means, or at least, that she means what she says, insisting that the two statements are the same thing. The numerous attendees of the tea party then correct her with a series of flipped sentences which have totally different meanings. For example, "I like what I get" and "I get what I like".

Are these two sentences saying the same thing? Can you think of other examples of flipped sentences?

The <mark>converse</mark> of a conditional statement written in the form "If *p*, then *q*" is the statement written in the form "If *q*, then *p*." The converse is a new statement that results when the hypothesis and conclusion of the conditional statement are interchanged.

The Corresponding Angle Postulate states: "If two parallel lines are intersected by a transversal, then the corresponding angles are congruent."

The **Corresponding Angle Converse Postulate** states: "If two lines intersected by a transversal form congruent corresponding angles, then the lines are parallel."

The Corresponding Angle Converse Postulate is used to prove new conjectures formed by writing the converses of the parallel lines theorems.

1. For each theorem:

   - Identify the hypothesis *p* and conclusion *q*.

   - Write the converse of the theorem as a conjecture.

   - State whether you think the converse conjecture is true or false.

   a. Alternate Interior Angle Theorem: If two parallel lines are intersected by a transversal, then the alternate interior angles are congruent.

   Hypothesis *p*:

   Conclusion *q*:

   Alternate Interior Angle Converse Conjecture: *If two lines intersected by a transversal form congruent alt. Int. ∡'s, then lines are //.*

   Truth Value of Converse Conjecture:

   b. Alternate Exterior Angle Theorem: If two parallel lines are intersected by a transversal, then the alternate exterior angles are congruent.

   Hypothesis *p*:

   Conclusion *q*:

   Alternate Exterior Angle Converse Conjecture: *If two lines intersected by a transversal form ≅ alt. ext. ∡'s, then lines are //.*

   Truth Value of Converse Conjecture:

   c. Same-Side Interior Angle Theorem: If two parallel lines are intersected by a transversal, then the same-side interior angles are supplementary.

   Hypothesis *p*:

   Conclusion *q*:

   Same-Side Interior Angle Converse Conjecture: *If two lines intersected by a transversal form supp. S.S. int. ∡'s, then lines are //.*

   Truth Value of Converse Conjecture:

d. Same-Side Exterior Angle Theorem: If two parallel lines are intersected by a transversal, then the same-side exterior angles are supplementary.

Hypothesis p:

Conclusion q:

Same-Side Exterior Angle Converse Conjecture: If two lines intersected by a transversal form supp. s.s. ext. ∡'s, then lines are ∥.

Truth Value of Converse Conjecture:

2. Consider the diagram with lines r and s, and point B.

a. Use the Corresponding Angle Converse Postulate to construct a line parallel to line r that passes through point B. Write the steps.
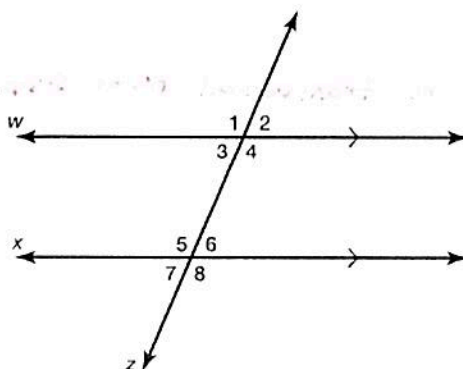

Snip
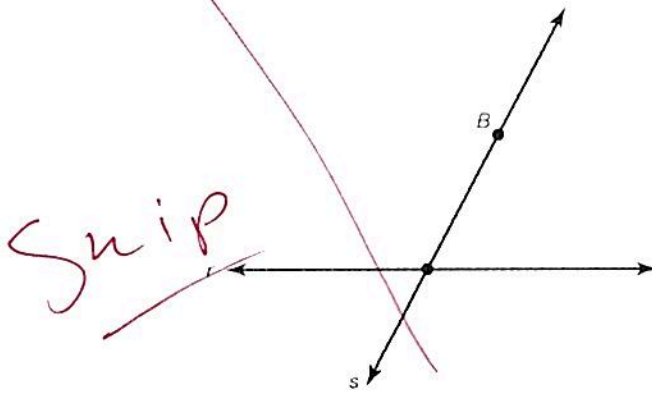
b. Which line is a transversal?

c. Which lines are parallel?

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1. The Alternate Interior Angle Converse Conjecture states: "If two lines intersected by a transversal form congruent alternate interior angles, then the lines are parallel."

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a. Use the diagram to write the given and prove statements for the Alternate Interior Angle Converse Conjecture.

Given:

Prove:

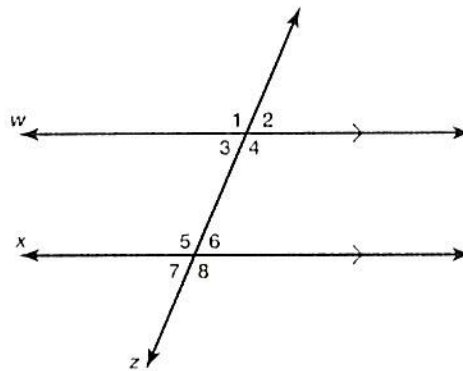b. Prove the Alternate Interior Angle Converse Conjecture.

Congratulations! You can now use this theorem as a valid reason in proofs.

You have just proven the Alternate Interior Angle Converse Conjecture. It is now known as the **Alternate Interior Angle Converse Theorem**.

4. The Same-Side Exterior Angle Converse Conjecture states: "If two lines intersected by a transversal form supplementary same-side exterior angles. then the lines are parallel."

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a. Use the diagram to write the given and prove statements for the Same-Side Exterior Angle Converse Conjecture.

Given:

Prove:

b. Prove the Same-Side Exterior Angle Converse Conjecture.

You have just proven the Same-Side Exterior Angle Converse Conjecture. It is now known as the **Same-Side Exterior Angle Converse Theorem**.

Here are all the converse postulates you have proven. Each converse conjecture you have proven is a new theorem.

Corresponding Angle Converse Postulate: If two lines intersected by a transversal form congruent corresponding angles, then the lines are parallel. <1 and <5

Alternate Interior Angle Converse Theorem: If two lines intersected by a transversal form congruent alternate interior angles, then the lines are parallel. <6 and <3

Alternate Exterior Angle Converse Theorem: If two lines intersected by a transversal form congruent alternate exterior angles, then the lines are parallel. <2 and <7

Same-Side Interior Angle Converse Theorem: If two lines intersected by a transversal form supplementary same-side interior angles, then the lines are parallel. <4 and <6

Same-Side Exterior Angle Converse Theorem: If two lines intersected by a transversal form supplementary same-side exterior angles, then the lines are parallel. <8 and <2

Use the diagram to answer the questions.
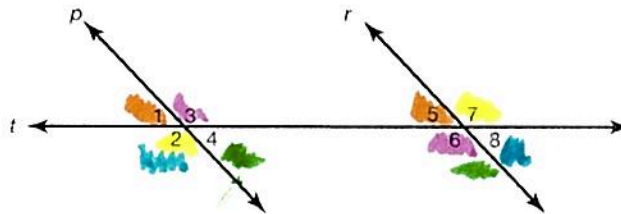
1. Which theorem or postulate would use $\angle 2 \cong \angle 7$ to justify line p is parallel to line r?

2. Which theorem or postulate would use $\angle 4 \cong \angle 5$ to justify line p is parallel to line r?

3. Which theorem or postulate would use $\angle 1 \cong \angle 5$ to justify line p is parallel to line r?