Coach C's Notes

of Proof

2.3

Paragraph Proof, Two-Column Proof, Construction Proof, and Flow Chart Proof

LEARNING GOALS

In this lesson, you will:

- Use the addition and subtraction properties of equality.
- Use the reflexive, substitution, and transitive properties.
- Write a paragraph proof.
- Prove theorems involving angles.
- Complete a two-column proof.
- Perform a construction proof.
- Complete a flow chart proof.

KEY TERMS

- Addition Property of Equality
- Subtraction Property of Equality
- Reflexive Property
- Substitution Property
- Transitive Property
- proof
- flow chart proof
- two-column proof
- paragraph proof
- construction proof
- Right Angle Congruence Theorem
- Congruent Supplement Theorem
- Congruent Complement Theorem
- Vertical Angle Theorem

Have you ever heard the famous phrase, "The proof is in the pudding"? If you stop to think about what this phrase means, you might be left scratching your head!

The phrase used now is actually a shortened version of the original phrase, "The proof of the pudding is in the eating." This phrase meant that the pudding recipe may appear to be delicious, include fresh ingredients, and may even look delicious after it is made. However, the only way to really "prove" that the pudding is delicious is by eating it!

Today it is used to imply that the quality or truth of something can only be determined by putting it into action. For example, you don't know how good an idea is until you actually test the idea.

Can you think of any other popular phrases that don't seem to make sense? Perhaps you should do a little research to find out where these phrases came from.

PROBLEM 1 Properties of Real Numbers in Geometry

Many properties of real numbers can be applied in geometry. These properties are important when making conjectures and proving new theorems.

The **Addition Property of Equality** states: "If a, b, and c are real numbers and $a = b$, then $a + c = b + c$."

The Addition Property of Equality can be applied to angle measures, segment measures, and distances.

Angle measures:

If $m\angle 1 = m\angle 2$, then $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$.

Segment measures:

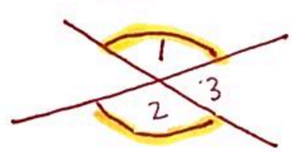
If $m\overline{AB} = m\overline{CD}$, then $m\overline{AB} + m\overline{EF} = m\overline{CD} + m\overline{EF}$.

Distances:

If $AB = CD$, then $AB + EF = CD + EF$.

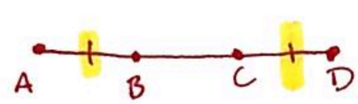
1. Sketch a diagram and write a statement that applies the Addition Property of Equality to angle measures.

If $m\angle 1 = m\angle 2$, then $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$



2. Sketch a diagram and write a statement that applies the Addition Property of Equality to segment measures.

If $m\overline{AB} = m\overline{CD}$, then $m\overline{AB} + m\overline{BC} = m\overline{CD} + m\overline{BC}$



The **Subtraction Property of Equality** states: "If a, b, and c are real numbers and $a = b$, then $a - c = b - c$."

The Subtraction Property of Equality can be applied to angle measures, segment measures, and distances.

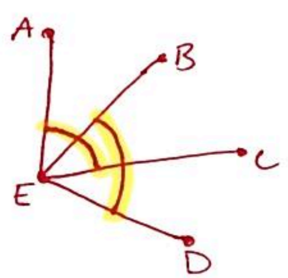
If $m\angle 1 = m\angle 2$, then $m\angle 1 - m\angle 3 = m\angle 2 - m\angle 3$.

If $m\overline{AB} = m\overline{CD}$, then $m\overline{AB} - m\overline{EF} = m\overline{CD} - m\overline{EF}$.

If $AB = CD$, then $AB - EF = CD - EF$.

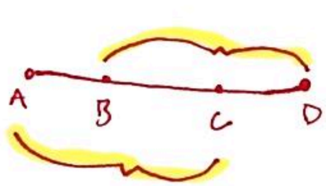
3. Sketch a diagram and write a statement that applies the Subtraction Property of Equality to angle measures.

If $m\angle AEC = m\angle BED$, then $m\angle AEC - m\angle BEC = m\angle BED - m\angle BEC$



4. Sketch a diagram and write a statement that applies the Subtraction Property of Equality to segment measures.

If $m\overline{AC} = m\overline{BD}$, then $m\overline{AC} - m\overline{BC} = m\overline{BD} - m\overline{BC}$



The **Reflexive Property** states: "If a is a real number, then $a = a$."

like looking in a mirror (it's the same) →

The Reflexive Property can be applied to angle measures, segment measures, distances, congruent angles, and congruent segments.

2

Angle measures:

$m\angle 1 = m\angle 1$

Segment measures:

$m\overline{AB} = m\overline{AB}$

Distances:

$AB = AB$

Congruent angles:

$\angle 1 \cong \angle 1$

Congruent segments:

$\overline{AB} \cong \overline{AB}$

5. Sketch a diagram and write a statement that applies the Reflexive Property to angles.

$\angle A = \angle A$

6. Sketch a diagram and write a statement that applies the Reflexive Property to segments.

$\overline{CD} \cong \overline{CD}$

The **Substitution Property** states: "If *a* and *b* are real numbers and $a = b$, then *a* can be substituted for *b*."

The Substitution Property can be applied to angle measures, segment measures, and distances.

If $m\angle 1 = 56°$ and $m\angle 2 = 56°$, then $m\angle 1 = m\angle 2$.

If $m\overline{AB} = 4$ mm and $m\overline{CD} = 4$ mm, then $m\overline{AB} = m\overline{CD}$.

If $AB = 12$ ft and $CD = 12$ ft, then $AB = CD$.

7. Sketch a diagram and write a statement that applies the Substitution Property to angles.

If $m\angle A = 90°$ and $m\angle B = 90°$, then $m\angle A = m\angle B$

8. Sketch a diagram and write a statement that applies the Substitution Property to segments.

If $m\overline{CD} = 8$ in. and $m\overline{EF} = 8$ in., then $m\overline{CD} = m\overline{EF}$

The **Transitive Property** states: "If a, b, and c are real numbers, $a = b$, and $b = c$, then $a = c$."

A special type of substitution. Like taking out the "middle man"

The Transitive Property can be applied to angle measures, segment measures, distances, congruent angles, and congruent segments.

Angle measures:

If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$.

Segment measures:

If $m\overline{AB} = m\overline{CD}$ and $m\overline{CD} = m\overline{EF}$, then $m\overline{AB} = m\overline{EF}$.

Distances:

If $AB = CD$ and $CD = EF$, then $AB = EF$.

Congruent angles:

If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.

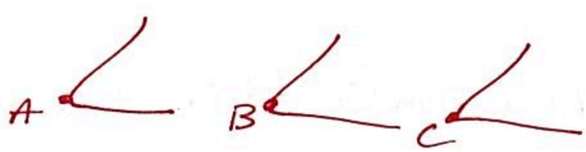
Congruent segments:

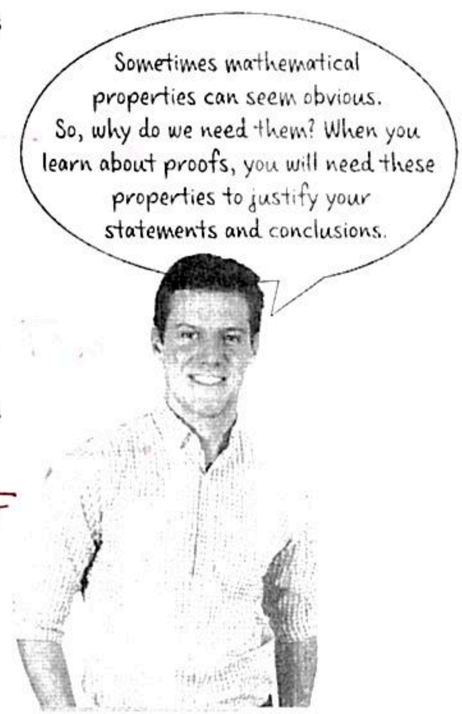
If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

"middle man"

9. Sketch a diagram and write a statement that applies the Transitive Property to angles.

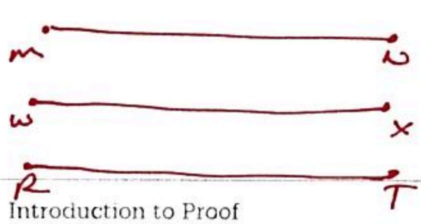
If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$





10. Sketch a diagram and write a statement that applies the Transitive Property to congruent segments.

If $\overline{MN} \cong \overline{WX}$ and $\overline{WX} \cong \overline{RT}$ then $\overline{MN} \cong \overline{RT}$



Various Forms of Proof

A **proof** is a logical series of statements and corresponding reasons that starts with a hypothesis and arrives at a conclusion. In this course, you will use four different kinds of proof.

1. The diagram shows four collinear points A, B, C, and D such that point B lies between points A and C, point C lies between points B and D, and $AB \cong CD$.

Consider the conditional statement: If $\overline{AB} \cong \overline{CD}$, then $\overline{AC} \cong \overline{BD}$.

a. Write the hypothesis as the "Given" and the conclusion as the "Prove."

Given: $\overline{AB} \cong \overline{CD}$

Prove: $AC \cong \overline{BD}$

A **flow chart proof** is a proof in which the steps and reasons for each step are written in boxes. Arrows connect the boxes and indicate how each step and reason is generated from one or more other steps and reasons.

Skip

b. Cut out the steps on the flow chart proof.

$\overline{AB} \cong \overline{CD}$ Given	$\overline{AC} \cong \overline{BD}$ Definition of congruent segments
$m\overline{AB} = m\overline{CD}$ Definition of congruent segments	$m\overline{BC} = m\overline{BC}$ Reflexive Property
$m\overline{AB} + m\overline{BC} = m\overline{CD} + m\overline{BC}$ Addition Property of Equality	$m\overline{AB} + m\overline{BC} = m\overline{AC}$ Segment Addition Postulate
$m\overline{AC} = m\overline{BD}$ Substitution Property	$m\overline{BC} + m\overline{CD} = m\overline{BD}$ Segment Addition Postulate

A **two-column proof** is a proof in which the steps are written in the left column and the corresponding reasons are written in the right column. Each step and corresponding reason are numbered.

d. Create a two-column proof of the conditional statement in Question 1. Each box of the flow chart proof in Question 1, part (c) should appear as a row in the two-column proof.

Given: $\overline{AB} \cong \overline{CD}$

Prove: $\overline{AC} \cong \overline{BD}$

what you know or conclude ↙ why you can conclude it. ↙

Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$	1. Given
2. $m\overline{AB} = m\overline{CD}$	2. Definition of congruent segments
3. $m\overline{BC} = m\overline{BC}$	3. Reflexive property
4. $m\overline{AB} + m\overline{BC} = m\overline{CD} + m\overline{BC}$	4. Addition property of equality
5. $m\overline{AB} + m\overline{BC} = m\overline{AC}$	5. Segment Addition Postulate
6. $m\overline{CD} + m\overline{BC} = m\overline{BD}$	6. Segment Addition Postulate
7. $m\overline{AC} = m\overline{BD}$	7. Substitution Property
8. $\overline{AC} \cong \overline{BD}$	8. Definition of congruent segments

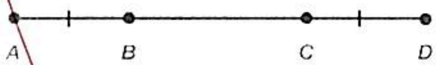
A **paragraph proof** is a proof in which the steps and corresponding reasons are written in complete sentences.

Skip

e. Write a paragraph proof of the conditional statement in Question 1. Each row of the two-column proof in Question 1, part (d) should appear as a sentence in the paragraph proof.

A **construction proof** is a proof that results from creating an object with specific properties using only a compass and a straightedge.

f. Create a proof by construction of the conditional statement in Question 2.



Given:

Prove:

Proof of the Right Angle Congruence Theorem

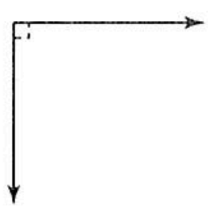


Fig. 1

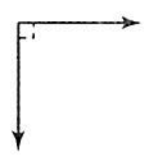
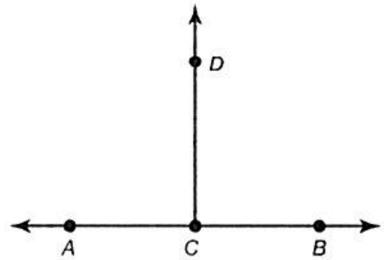


Fig. 2

1. Karl insists the angle in Figure 1 is larger than the angle in Figure 2. Roy disagrees and insists both angles are the same size. Who is correct? What is your reasoning?

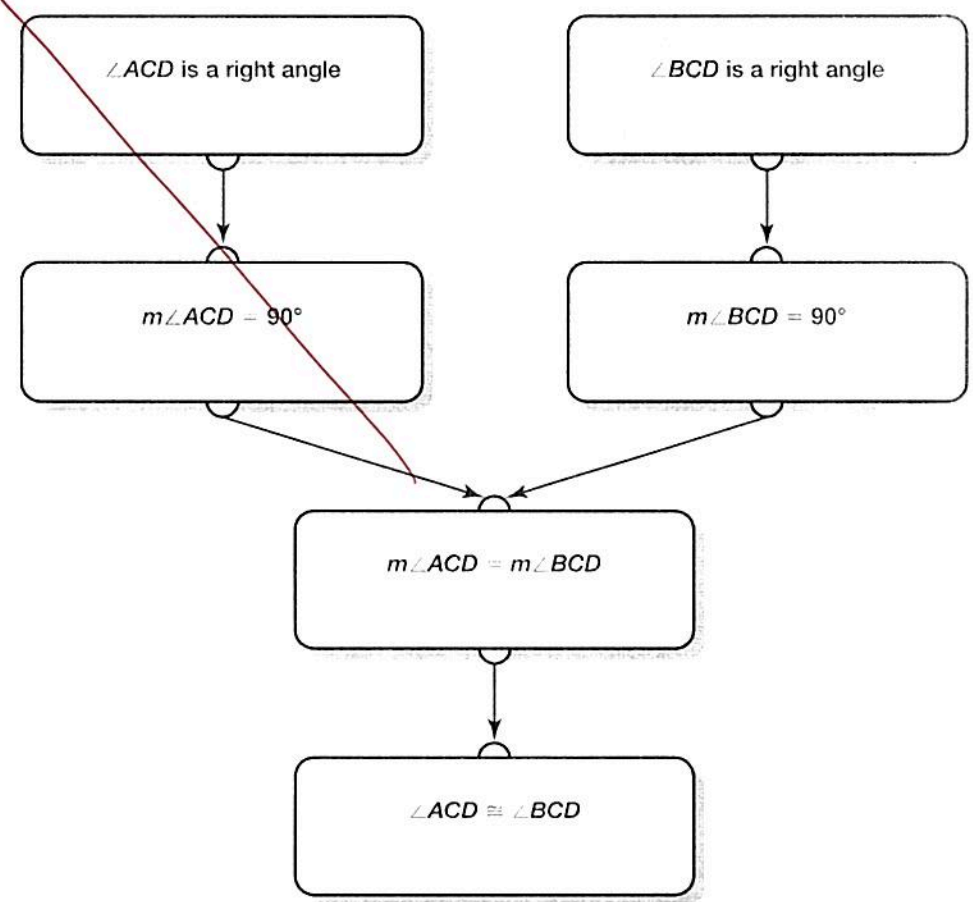
The **Right Angle Congruence Theorem** states: "All right angles are congruent."



Given: $\angle ACD$ and $\angle BCD$ are right angles.

Prove: $\angle ACD \cong \angle BCD$

Complete the flow chart of the Right Angle Congruence Theorem by writing the statement for each reason in the boxes provided.



Proofs of the Congruent Supplement Theorem

The **Congruent Supplement Theorem** states: "If two angles are supplements of the same angle or of congruent angles, then the angles are congruent."

1. Use the diagram to write the "Given" statements for the Congruent Supplement Theorem. The "Prove" statement is provided.

   Given: ∠1 is supplement to ∠2

   Given: ∠3 is supplement to ∠4

   Given: ∠2 ≅ ∠4

   Prove: ∠1 ≅ ∠3

Skip

2. Cut out the [illegible] of the flow chart proof.

∠1 is supp[illegible]entary to ∠2 G[illegible]	∠3 is supplementary to ∠4 Given
$m\angle 1 + m\angle 2 = 180°$ Definition of supplementary angles	∠2 ≅ ∠4 Given
$m\angle 3 + m\angle 4 = 180°$ Definition of supplementary angles	$m\angle 1 = m\angle 3$ Subtraction Property of Equality
$m\angle 2 = m\angle 4$ Definition of congruent angles	$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$ Substitution Property
∠1 ≅ ∠3 Definition of congruent angles	

4. Create a two-column proof of the Congruent Supplement Theorem.
Each box of the flow chart proof in Question 3 should appear as a row in the two-column proof.

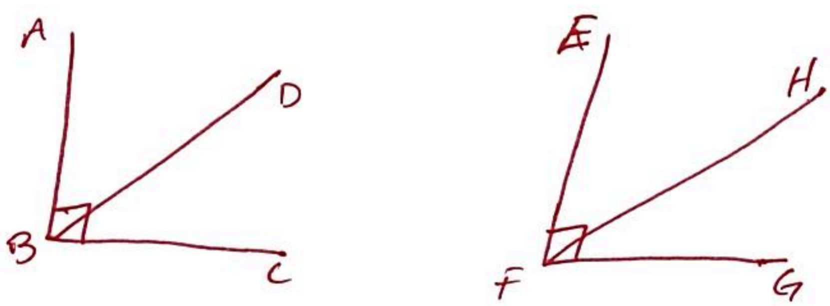
Statements	Reasons
1. ∠1 is supplement to ∠2	1. Given
2. ∠3 is supplement to ∠4	2. Given
3. ∠2 ≅ ∠4	3. Given
4. m∠2 = m∠4	4. Def. ≅ ∠'s
5. [redacted] m∠1 + m∠2 = 180°	5. Def. supplementary ∠'s
6. m∠3 + m∠4 = 180°	6. Def. Supplementary ∠'s
7. m∠1 + m∠2 = m∠3 + m∠4	7. Substitution
8. m∠1 = m∠3	8. Transitive
9. ∠1 ≅ ∠3	9. Def. ≅ ∠'s

2

PROBLEM 5 Proofs of the Congruent Complement Theorem

The **Congruent Complement Theorem** states: "If two angles are complements of the same angle or of congruent angles, then they are congruent."

1. Draw and label a diagram illustrating this theorem.



2. Use your diagram to write the "Given" and "Prove" statements for the Congruent Complement Theorem.

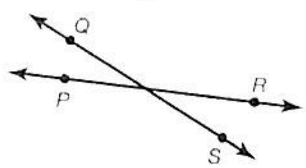
Given: ∠DBC is comolement to ∠ABD
Given: ∠HFG is complement to ∠EFH
Given: ∠ABD ≅ ∠EFH
Prove: ∠DBC ≅ ∠HFG

4. Create a two-column proof of the Congruent Complement Theorem. Each box of the flow chart proof in Question 3 should appear as a row in the two-column proof.

Statements	Reasons
Same as last proof but as	complementary angles
(Extra practice)	

Proofs of the Vertical Angle Theorem

The diagram shows the intersection of two lines, $\overleftrightarrow{PR}$ and $\overleftrightarrow{QS}$. The intersection of these lines forms two pairs of vertical angles. Vertical angles can be formed in other ways as well:

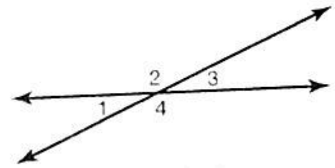


Skip

- by the intersection of two line segments ($\overline{PR}$ and $\overline{QS}$)
- by the intersection of two rays ($\overrightarrow{PR}$ and $\overrightarrow{QS}$)
- by the intersection of a line segment and a ray ($\overline{PR}$ and $\overrightarrow{QS}$)

1. What other ways can vertical angles be formed?

2. Use construction tools to verify that vertical angles are congruent.



a. Identify the angle that forms a vertical angle pair with ∠1. Explain your reasoning.

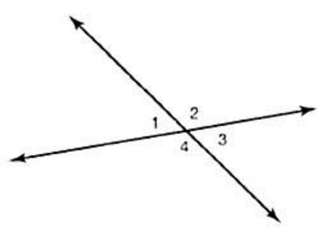
b. Use construction tools to duplicate ∠1. Use point *A* on the given starter line as the vertex of the angle.

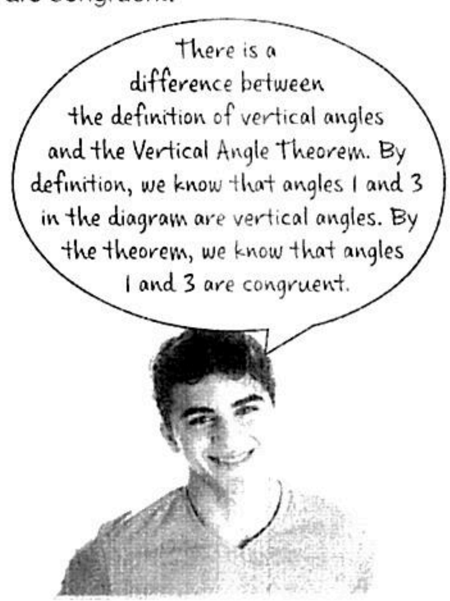
c. Use construction tools to duplicate ∠3. Use point *A* on the starter line as the vertex of the angle.

d. What can you conclude about angles 1 and 3? Explain your reasoning.

$\angle 1 \cong \angle 3$

The **Vertical Angle Theorem** states: "Vertical angles are congruent."





3. Use the diagram to write the "Prove" statements for the Vertical Angle Theorem. The "Given" statements are provided.

Given: ∠1 and ∠2 are a linear pair

Given: ∠2 and ∠3 are a linear pair

Given: ∠3 and ∠4 are a linear pair

Given: ∠4 and ∠1 are a linear pair

Prove: $\angle 1 \cong \angle 3$

Prove: $\angle 2 \cong \angle 4$

4. Create a flow chart proof of the first "Prove" statement of the Vertical Angle Theorem.

Skip

5. Create a two-column proof of the second "Prove" statement of the Vertical Angle Theorem.

Given: ∠4 and ∠1 are a linear pair

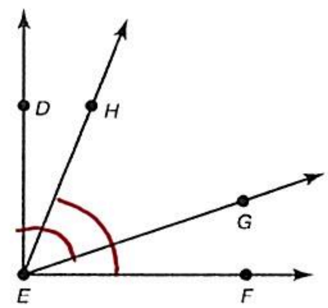
Given: ∠1 and ∠2 are a linear pair

Prove: ∠2 ≅ 4

Statements	Reasons
1. ∠4 + ∠1 linear pair	1. Given
2. ∠1 + ∠2 linear pair	2. Given
3. ∠4 + ∠1 supplementary ∠'s	3. Linear Pair postulate
4. ∠1 + ∠2 supplementary ∠'s	4. Linear Pair postulate
5. m∠4 + m∠1 = 180°	5. Def. supplementary ∠'s
6. m∠1 + m∠2 = 180°	6. Def. Supplementary ∠'s
7. m∠4 + m∠1 = m∠1 + m∠2	7. Substitution property
8. m∠4 = m∠2	8. Transitive property
9. ∠4 ≅ ∠2	9. Def. ≅ ∠'s

PROBLEM 7 Proofs Using the Angle Addition Postulate

Given: ∠DEG ≅ ∠HEF
Prove: ∠DEH ≅ ∠GEF



1. ~~Prove~~ the conditional statement using ~~any method you choose~~.
2-column Proof

Statements	Reasons
1. ∠DEG ≅ ∠HEF	1. Given
2. m∠DEG = m∠HEF	2. Def. ≅ ∡'s
3. m∠DEH + m∠HEG = m∠DEG	3. ∡ add. postulate
4. m∠GEF + m∠HEG = m∠HEF	4. ∡ add. postulate
5. m∠HEG = m∠HEG	5. Reflexive property
6. m∠DEH + m∠HEG = m∠GEF + m∠HEG	6. substitution
7. m∠DEH = m∠GEF	7. Transitive
8. ∠DEH ≅ ∠GEF	8. Def. ≅ ∡'s

2