Properties of Triangles Notes

Classifying Triangles:

Angles

a. right triangle

one $\measuredangle = 90^\circ$

b. acute triangle

all $\measuredangle$'s less than 90°

c. obtuse triangle

one $\measuredangle$ greater than 90°

Sides

a. equilateral triangle

all sides same

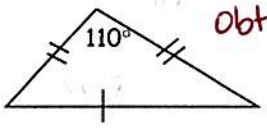
b. isosceles triangle

at least two sides same

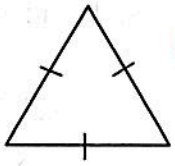
c. scalene triangle

no sides the same

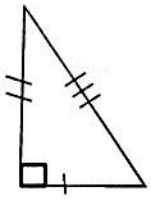
Naming Triangles:



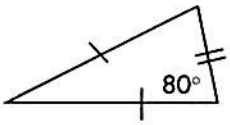
Obtuse isosceles



acute equilateral



right scalene



acute isosceles

Plot A (0, -4), B (0, -9), and C (-2, -5). Classify the triangle by its sides and angles.

$AB = -4 - (-9)$

$= 5$

$BC = \sqrt{(-2-0)^2 + (-5-(-9))^2}$

$= \sqrt{(-2)^2 + (4)^2}$

$= \sqrt{4+16}$

$= \sqrt{20}$

$AC = \sqrt{(-2-0)^2 + (-5-(-4))^2}$

$= \sqrt{(-2)^2 + (-1)^2}$

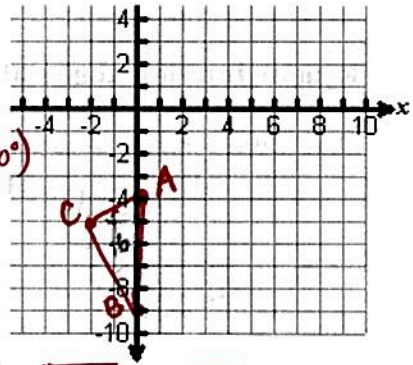
$= \sqrt{4+1}$

Slope

$AC = \frac{1}{2}$

$BC = \frac{-4}{2} = -2$

$\perp$ = right (90°)



Sides: $5 \neq \sqrt{20} \neq \sqrt{5}$

angle: one 90°

Triangle: right scalene

Triangle Sum Theorem:

Definition: the sum of the measures of the interior angles of a triangle is 180°

1. If the measure of the interior angles of a triangle are 57°, 62°, and 61°, descried the location of each side of the triangle without drawing a picture.

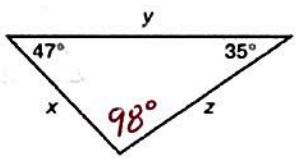
   a. longest side opposite 62° angle

   b. shortest side opposite 57° angle

2. When an angle of a triangle increases in measure, what happens to each side of the triangle? the sides adjacent to that angle stay the same, the side opposite that angle increases in length.

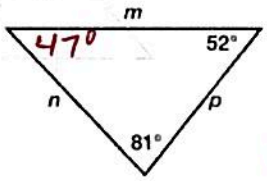
3. List the sides from shortest to longest for each diagram.

a.



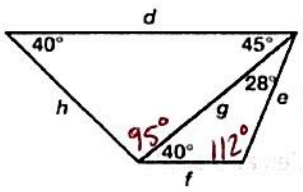
X, Z, Y

b.



P, n, m

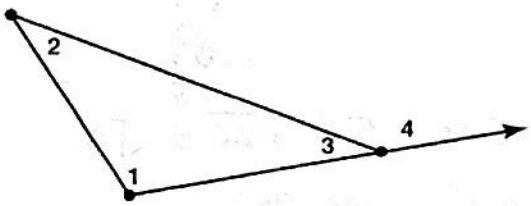
c.



f, e, g, h, d

Remote Interior Angles of a Triangle:

Definition: the two angles that are non-adjacent to the specified exterior angle.



Interior angles: ∠1, ∠2, ∠3

Exterior angle: ∠4

Remote interior angles: ∠1, ∠2

Exterior Angle Theorem:

Definition: the measure of the exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles of the triangle.

Exterior Angle Inequality Theorem:

Definition: the measure of an exterior angle of a triangle is greater than the measure of either of the remote interior angles.

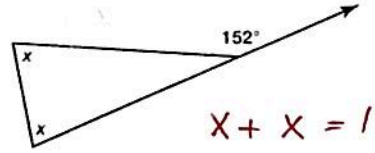
Solve for x in each diagram:

a.

$108 + x = 156$

$x = 48$

b.

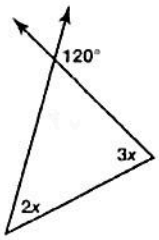


$x + x = 152$

$2x = 152$

$x = 76$

c.

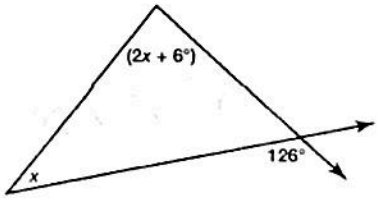


$2x + 3x = 120$

$5x = 120$

$x = 24$

d.



$x + (2x + 6) = 126$

$3x + 6 = 126$

$3x = 120$

$x = 40$

Triangle Inequality Theorem:

Definition: the sum of lengths of any two sides of a triangle is greater than the length of the third side.

Apply the Triangle Inequality Theorem to determine whether the side lengths listed can form a triangle:

a. 24 in., 24 in., 16 in.
yes $24 + 16 = 40$ $40 > 24$ ✓

b. 25in.. 52 in., 26 in.
No $25 + 26 = 51$ $51 < 52$ X

c. 40 in., 40 in., 40 in.
yes $40 + 40 = 80$ $80 > 40$ ✓

d. 45 in.. 31 in., 75 in.
yes $45 + 31 = 76$ $76 > 75$ ✓

If the two sides of a triangle are 4 and 5. Find the range of values for the third side

5 could be longest

$4 + X > 5$

$X > 1$

X could be longest

$X < 4 + 5$

$X < 9$

★ range: $1 < X < 9$ ★