Carnegie Textbook

Glossary

Glossary

A

Addition Property of Equality

The addition property of equality states:
"If $a = b$, then $a + c = b + c$."

Example

If $x = 2$, then $x + 5 = 2 + 5$, or $x + 5 = 7$ is an example of the Addition Property of Equality.

Addition Rule for Probability

The Addition Rule for Probability states: "The probability that Event A occurs or Event B occurs is the probability that Event A occurs plus the probability that Event B occurs minus the probability that both A and B occur."

$$P(A \text{ or } B) = P(A) + P(B) = P(A \text{ and } B)$$

Example

You flip a coin two times. Calculate the probability of flipping a heads on the first flip or flipping a heads on the second flip.

Let A represent the event of flipping a heads on the first flip. Let B represent the event of flipping a heads on the second flip.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$
$$P(A \text{ or } B) = \frac{1}{2} + \frac{1}{2} - \frac{1}{4}$$
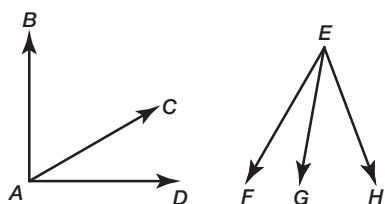$$P(A \text{ or } B) = \frac{3}{4}$$

So, the probability of flipping a heads on the first flip or flipping a heads on the second flip is $\frac{3}{4}$.

adjacent angles

Adjacent angles are angles that share a common side and a common vertex, and lie on opposite sides of their common side.

Example

Angle BAC and angle CAD are adjacent angles.
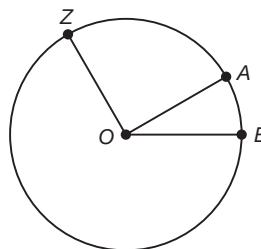Angle FEG and angle GEH are adjacent angles.

adjacent arcs

Adjacent arcs are two arcs of the same circle sharing a common endpoint.
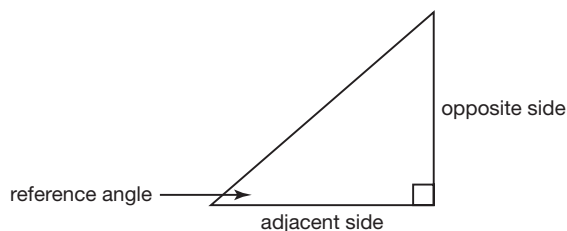
Example

Arcs ZA and AB are adjacent arcs.

adjacent side

The adjacent side of a triangle is the side adjacent to the reference angle that is not the hypotenuse.
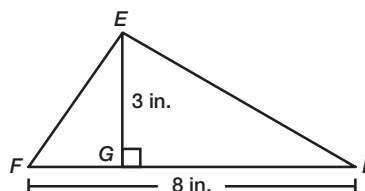
Example

altitude

An altitude is a line segment drawn from a vertex of a triangle perpendicular to the line containing the opposite side.

Example

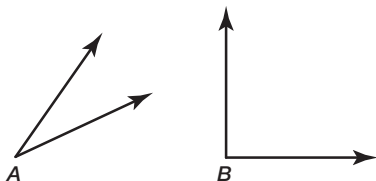Segment EG is an altitude of triangle FED.

angle

An angle is a figure that is formed by two rays that extend from a common point called the vertex.

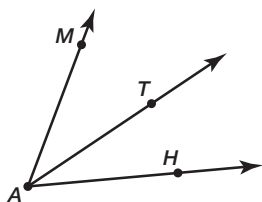**Example**

Angles *A* and *B* are shown.

angle bisector

An angle bisector is a ray that divides an angle into two angles of equal measure.

Example

Ray *AT* is the angle bisector of angle *MAH*.

angular velocity

Angular velocity is a type of circular velocity described as an amount of angle movement in radians over a specified amount of time. Angular velocity can be expressed as $\omega = \frac{\theta}{t}$, where $\omega =$ angular velocity, $\theta =$ angular measurement in radians, and $t =$ time.

annulus

An annulus is the region bounded by two concentric circles.

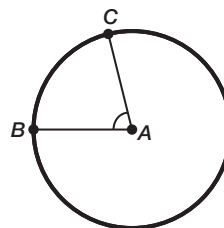**Example**

The annulus is the shaded region shown.
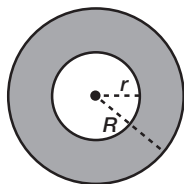
arc

An arc is the curve between two points on a circle. An arc is named using its two endpoints.

Example

The symbol used to describe arc *BC* is $\overset{\frown}{BC}$.

arc length

An arc length is a portion of the circumference of a circle. The length of an arc of a circle can be calculated by multiplying the circumference of the circle by the ratio of the measure of the arc to 360°.
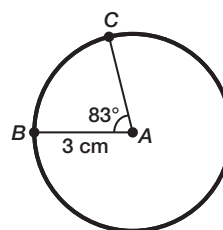
$$\text{arc length} = 2\pi r \cdot \frac{x°}{360°}$$

Example

In circle *A,* the radius $\overline{AB}$ is 3 centimeters and the measure of arc *BC* is 83 degrees.

$$(2\pi r)\left(\frac{m\overset{\frown}{BC}}{360°}\right) = 2\pi(3)\left(\frac{83°}{360°}\right)$$
$$\approx 4.35$$

So, the length of arc *BC* is approximately 4.35 centimeters.
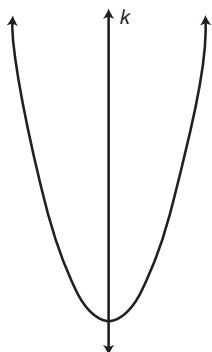
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axis of symmetry

An axis of symmetry is a line that passes through a figure and divides the figure into two symmetrical parts that are mirror images of each other.

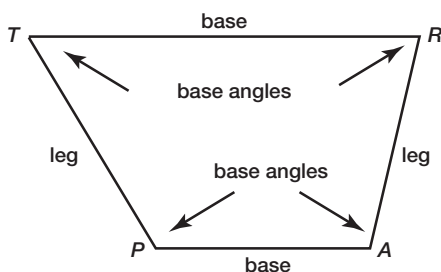**Example**

Line *k* is the axis of symmetry of the parabola.

B

base angles of a trapezoid

The base angles of a trapezoid are either pair of angles that share a base as a common side.

Example

Angle *T* and angle *R* are one pair of base angles of trapezoid *PART*. Angle *P* and angle *A* are another pair of base angles.
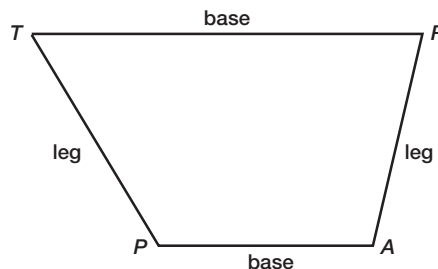

bases of a trapezoid

The parallel sides of a trapezoid are the bases of the trapezoid.

Example

Line segment *TR* and line segment *PA* are the bases of trapezoid *PART*.

biconditional statement

A biconditional statement is a statement written in the form "if and only if *p*, then *q*." It is a combination of both a conditional statement and the converse of that conditional statement. A biconditional statement is true only when the conditional statement and the converse of the statement are both true.

Example

Consider the property of an isosceles trapezoid: "The diagonals of an isosceles trapezoid are congruent." The property states that if a trapezoid is isosceles, then the diagonals are congruent. The converse of this statement is true: "If the diagonals of a trapezoid are congruent, then the trapezoid is an isosceles trapezoid." So, this property can be written as a biconditional statement: "A trapezoid is isosceles if and only if its diagonals are congruent."

C

categorical data (qualitative data)

Categorical data are data that each fit into exactly one of several different groups, or categories. Categorical data are also called "qualitative data."

Example

Animals: lions, tigers, bears, etc.
U.S. Cities: Los Angeles, Atlanta, New York City, Dodge City, etc.

The set of animals and the set of U.S. cities are two examples of categorical data sets.

Glossary

Cavalieri's principle

Cavalieri's principle states that if all one-dimensional slices of two-dimensional figures have the same lengths, then the two-dimensional figures have the same area. The principle also states that given two solid figures included between parallel planes, if every plane cross section parallel to the given planes has the same area in both solids, then the volumes of the solids are equal.

center of a circle

The center of a circle is a fixed point in the plane that is at an equal distance from every point on the circle.

Example
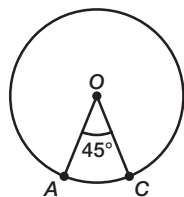
Point *H* is the center of the circle.
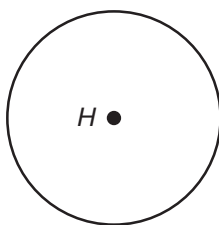
central angle

A central angle of a circle is an angle whose sides are radii. The measure of a central angle is equal to the measure of its intercepted arc.

Example

In circle *O*, $\angle AOC$ is a central angle and $\overset{\frown}{AC}$ is its intercepted arc. If $m\angle AOC = 45°$, then $m\overset{\frown}{AC} = 45°$.
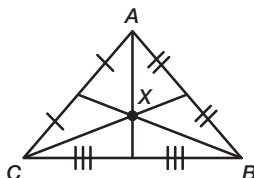
centroid

The centroid of a triangle is the point at which the medians of the triangle intersect.

Example

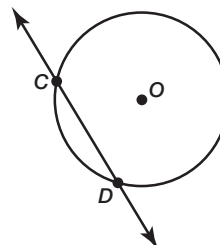Point *X* is the centroid of triangle *ABC*.

chord

A chord is a line segment whose endpoints are points on a circle. A chord is formed by the intersection of the circle and a secant line.

Example

Segment *CD* is a chord of circle *O*.

circular permutation

A circular permutation is a permutation in which there is no starting point and no ending point. The circular permutation of *n* objects is $(n - 1)!$.
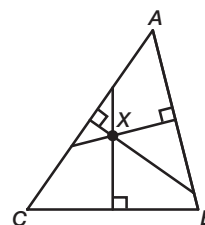
Example

A club consists of four officers: a president (P), a vice-president (VP), a secretary (S), and a treasurer (T). There are $(4 - 1)!$, or 6 ways for the officers to sit around a round table.

circumcenter

The circumcenter of a triangle is the point at which the perpendicular bisectors intersect.

Example

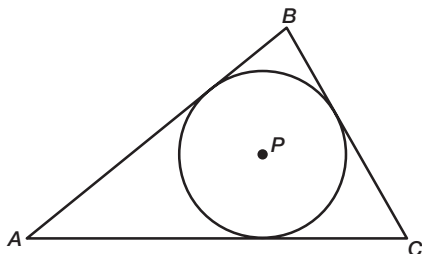Point *X* is the circumcenter of triangle *ABC*.

circumscribed polygon

A circumscribed polygon is a polygon drawn outside a circle such that each side of the polygon is tangent to the circle.
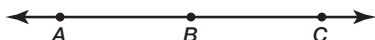
Example

Triangle *ABC* is a circumscribed triangle.

collinear points

Collinear points are points that are located on the same line.

Example

Points *A*, *B*, and *C* are collinear.

combination

A combination is an unordered collection of items. One notation for the combinations of *r* elements taken from a collection of *n* elements is:

$$_nC_r = C(n, r) = C^n_r$$

Example

The two-letter combinations of the letters *A*, *B*, and *C* are: *AB*, *AC*, *BC*.

compass

A compass is a tool used to create arcs and circles.

Example

complement of an event

The complement of an event is an event that contains all the outcomes in the sample space that are not outcomes in the event. In mathematical notation, if *E* is an event, then the complement of *E* is often denoted as $\overline{E}$ or E^c.

Example

A number cube contains the numbers 1 though 6. Let *E* represent the event of rolling an even number. The complement of Event *E* is rolling an odd number.
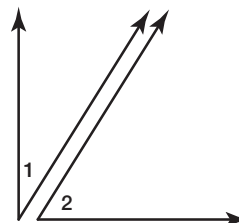
complementary angles

Two angles are complementary if the sum of their measures is 90º.

Example

Angle 1 and angle 2 are complementary angles.
$m\angle 1 + m\angle 2 = 90°$

composite figure

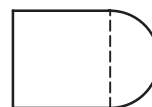A composite figure is formed by combining different shapes.

Example

The composite figure shown is formed by a square and a semicircle.

compound event

A compound event combines two or more events, using the word "and" or the word "or."

Example

You roll a number cube twice. Rolling a six on the first roll and rolling an odd number on the second roll are compound events.

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concavity

The concavity of a parabola describes the orientation of the curvature of the parabola.

Example

concave up

concave right
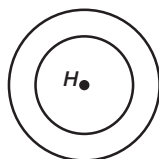

concave down

concave left
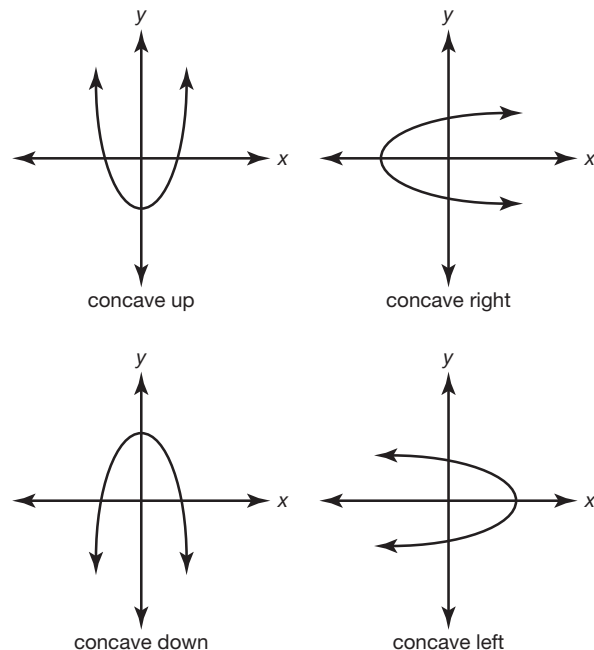
concentric circles

Concentric circles are circles in the same plane that have a common center.

Example

The circles shown are concentric because they are in the same plane and have a common center *H*.

conclusion

Conditional statements are made up of two parts. The conclusion is the result that follows from the given information.
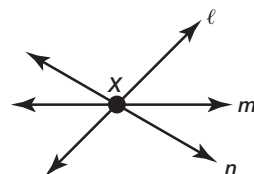
Example

In the conditional statement "If two positive numbers are added, then the sum is positive," the conclusion is "the sum is positive."

concurrent

Concurrent lines, rays, or line segments are three or more lines, rays, or line segments intersecting at a single point.

Example

Lines ℓ, *m*, and *n* are concurrent lines.

conditional probability

A conditional probability is the probability of event *B*, given that event *A* has already occurred. The notation for conditional probability is $P(B|A)$, which reads, "the probability of event *B*, given event *A*."

Example

The probability of rolling a 4 or less on the second roll of a number cube, given that a 5 is rolled first, is an example of a conditional probability.

conditional statement

A conditional statement is a statement that can be written in the form "If *p*, then *q*."

Example

The statement "If I close my eyes, then I will fall asleep" is a conditional statement.

congruent line segments

Congruent line segments are two or more line segments that have equal measures.

Example

Line segment *AB* is congruent to line segment *CD*.

conjecture

A conjecture is a hypothesis that something is true. The hypothesis can later be proved or disproved.
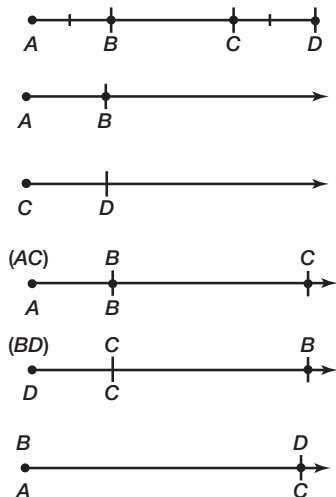
construct

A constructed geometric figure is created using only a compass and a straightedge.

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construction proof

A construction proof is a proof that results from creating a figure with specific properties using only a compass and straightedge.

Example

A construction proof is shown of the conditional statement: If $\overline{AB} \cong \overline{CD}$, then $\overline{AC} \cong \overline{BD}$.

contrapositive

To state the contrapositive of a conditional statement, negate both the hypothesis and the conclusion and then interchange them.

    Conditional Statement: If p, then q.
    Contrapositive: If not q, then not p.

Example

Conditional Statement: If a triangle is equilateral, then it is isosceles.

Contrapositive: If a triangle is not isosceles, then it is not equilateral.

converse

To state the converse of a conditional statement, interchange the hypothesis and the conclusion.

    Conditional Statement: If p, then q.
    Converse: If q, then p.

Example

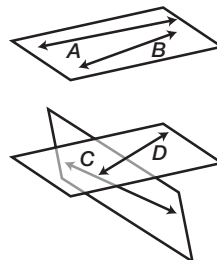Conditional Statement: If $a = 0$ or $b = 0$, then $ab = 0$.
Converse: If $ab = 0$, then $a = 0$ or $b = 0$.

coplanar lines

Coplanar lines are lines that lie in the same plane.

Example

Line A and line B are coplanar lines. Line C and line D are not coplanar lines.

corresponding parts of congruent triangles are congruent (CPCTC)

CPCTC states that if two triangles are congruent, then each part of one triangle is congruent to the corresponding part of the other triangle.

Example

In the triangles shown, $\triangle XYZ \cong \triangle LMN$. Because corresponding parts of congruent triangles are congruent (CPCTC), the following corresponding parts are congruent.

- $\angle X \cong \angle L$
- $\angle Y \cong \angle M$
- $\angle Z \cong \angle N$
- $\overline{XY} \cong \overline{LM}$
- $\overline{YZ} \cong \overline{MN}$
- $\overline{XZ} \cong \overline{LN}$
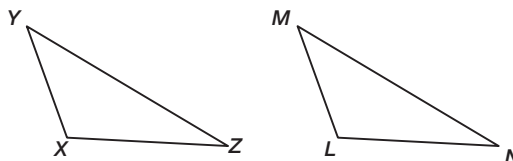
cosecant (csc)
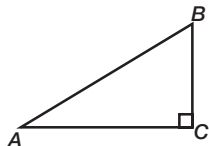
The cosecant (csc) of an acute angle in a right triangle is the ratio of the length of the hypotenuse to the length of the side opposite the angle.

Example

In triangle *ABC*, the cosecant of angle *A* is:

$$\csc A = \frac{\text{length of hypotenuse}}{\text{length of side opposite } \angle A} = \frac{AB}{BC}$$

The expression "csc *A*" means "the cosecant of angle *A*."
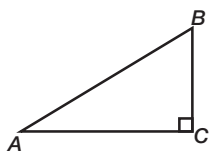

cosine (cos)

The cosine (cos) of an acute angle in a right triangle is the ratio of the length of the side adjacent to the angle to the length of the hypotenuse.

Example

In triangle *ABC*, the cosine of angle *A* is:

$$\cos A = \frac{\text{length of side adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{AC}{AB}$$

The expression "cos *A*" means "the cosine of angle *A*."
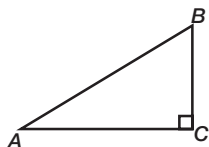

cotangent (cot)

The cotangent (cot) of an acute angle in a right triangle is the ratio of the length of the side adjacent to the angle to the length of the side opposite the angle.

Example

In triangle *ABC*, the cotangent of angle *A* is:

$$\cot A = \frac{\text{length of side adjacent to } \angle A}{\text{length of side opposite } \angle A} = \frac{AC}{BC}$$

The expression "cot *A*" means "the cotangent of angle *A*."

counterexample

A counterexample is a single example that shows that a statement is not true.

Example

Your friend claims that you add fractions by adding the numerators and then adding the denominators. A counterexample is $\frac{1}{2} + \frac{1}{2}$. The sum of these two fractions is 1. Your friend's method results in $\frac{1+1}{2+2}$, or $\frac{1}{2}$. Your friend's method is incorrect.

Counting Principle

The Counting Principle states that if action *A* can occur in *m* ways and for each of these *m* ways action *B* can occur in *n* ways, then actions *A* and *B* can occur in *m · n* ways.

Example

In the school cafeteria, there are 3 different main entrées and 4 different sides. So, there are 3 · 4, or 12 different lunches that can be created.

D

deduction

Deduction is reasoning that involves using a general rule to make a conclusion.
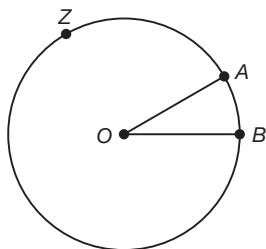
Example

Sandy learned the rule that the sum of the measures of the three interior angles of a triangle is 180 degrees. When presented with a triangle, she concludes that the sum of the measures of the three interior angles is 180 degrees. Sandy reached the conclusion using deduction.

Glossary

degree measure of an arc

The degree measure of a minor arc is equal to the degree measure of its central angle. The degree measure of a major arc is determined by subtracting the degree measure of the minor arc from 360°.

Example

The measure of minor arc AB is 30°. The measure of major arc BZA is 360° − 30° = 330°.

dependent events

Dependent events are events for which the occurrence of one event has an impact on the occurrence of subsequent events.
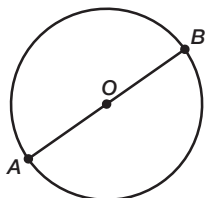
Example

A jar contains 1 blue marble, 1 green marble, and 2 yellow marbles. You randomly choose a yellow marble without replacing the marble in the jar, and then randomly choose a yellow marble again. The events of randomly choosing a yellow marble first and randomly choosing a yellow marble second are dependent events because the 1st yellow marble was not replaced in the jar.

diameter

The diameter of a circle is a line segment with each endpoint on the circle that passes through the center of the circle.
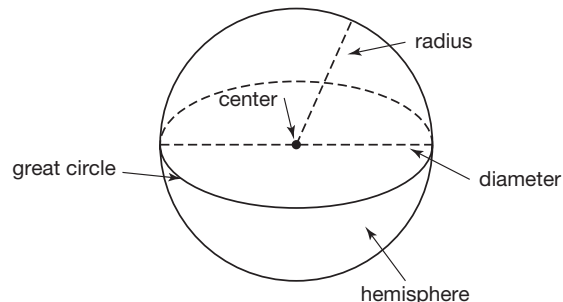
Example

In circle O, $\overline{AB}$ is a diameter.

diameter of a sphere

The diameter of a sphere is a line segment with each endpoint on the sphere that passes through the center of the sphere.

Example

direct proof

A direct proof begins with the given information and works to the desired conclusion directly through the use of givens, definitions, properties, postulates, and theorems.

directrix of a parabola

The directrix of a parabola is a line such that all points on the parabola are equidistant from the focus and the directrix.

Example

The focus of the parabola shown is the point (0, 2). The directrix of the parabola shown is the line $y = -2$. All points on the parabola are equidistant from the focus and the directrix.
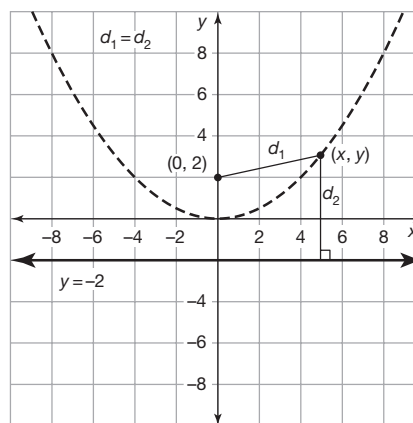
disc

A disc is the set of all points on a circle and in the interior of a circle.

disjoint sets

Two or more sets are disjoint sets if they do not have any common elements.

Example

Let N represent the set of 9th grade students. Let T represent the set of 10th grade students. The sets N and T are disjoint sets because the two sets do not have any common elements. Any student can be in one grade only.

Distance Formula

The Distance Formula can be used to calculate the distance between two points.

The distance between points (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Example

To calculate the distance between the points $(-1, 4)$ and $(2, -5)$, substitute the coordinates into the Distance Formula.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(2 + 1)^2 + (-5 - 4)^2}$

$d = \sqrt{3^2 + (-9)^2}$

$d = \sqrt{9 + 81}$

$d = \sqrt{90}$

$d \approx 9.49$

So, the distance between the points $(-1, 4)$ and $(2, -5)$ is approximately 9.49 units.

draw

To draw is to create a geometric figure using tools such as a ruler, straightedge, compass, or protractor. A drawing is more accurate than a sketch.

E

element

A member of a set is called an element of that set.

Example

Set B contains the elements a, b, and c.

$B = \{a, b, c\}$

endpoint of a ray

An endpoint of a ray is a point at which a ray begins.

Example

Point C is the endpoint of ray CD.

endpoints of a line segment

An endpoint of a line segment is a point at which a segment begins or ends.

Examples

Points A and B are endpoints of segment AB.

Glossary

Euclidean geometry

Euclidean geometry is a complete system of geometry developed from the work of the Greek mathematician Euclid. He used a small number of undefined terms and postulates to systematically prove many theorems.

Euclid's first five postulates are:

1. A straight line segment can be drawn joining any two points.
2. Any straight line segment can be extended indefinitely in a straight line.
3. Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.
4. All right angles are congruent.
5. If two lines are drawn that intersect a third line in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough. (This postulate is equivalent to what is known as the parallel postulate.)

Example

Euclidean geometry

Non-Euclidean geometry
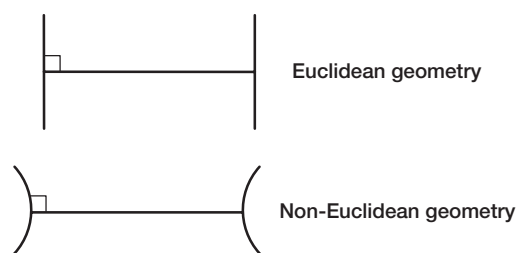
event

An event is an outcome or a set of outcomes in a sample space.

Example

A number cube contains the numbers 1 through 6. Rolling a 6 is one event. Rolling an even number is another event.

expected value

The expected value is the average value when the number of trials in a probability experiment is large.

experimental probability

Experimental probability is the ratio of the number of times an event occurs to the total number of trials performed.

Example

You flip a coin 100 times. Heads comes up 53 times. The experimental probability of getting heads is $\frac{53}{100}$.
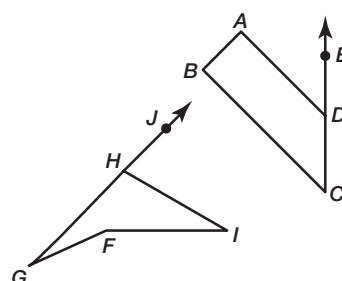
exterior angle of a polygon

An exterior angle of a polygon is an angle that is adjacent to an interior angle of a polygon.

Examples

Angle *JHI* is an exterior angle of quadrilateral *FGHI*.

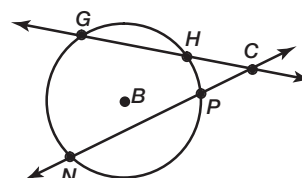Angle *EDA* is an exterior angle of quadrilateral *ABCD*.

external secant segment

An external secant segment is the portion of each secant segment that lies outside of the circle. It begins at the point at which the two secants intersect and ends at the point where the secant enters the circle.

Example

Segment *HC* and segment *PC* are external secant segments.

F

factorial

The factorial of *n*, written as *n*!, is the product of all non-negative integers less than or equal to *n*.

Example

$3! = 3 \times 2 \times 1 = 6$

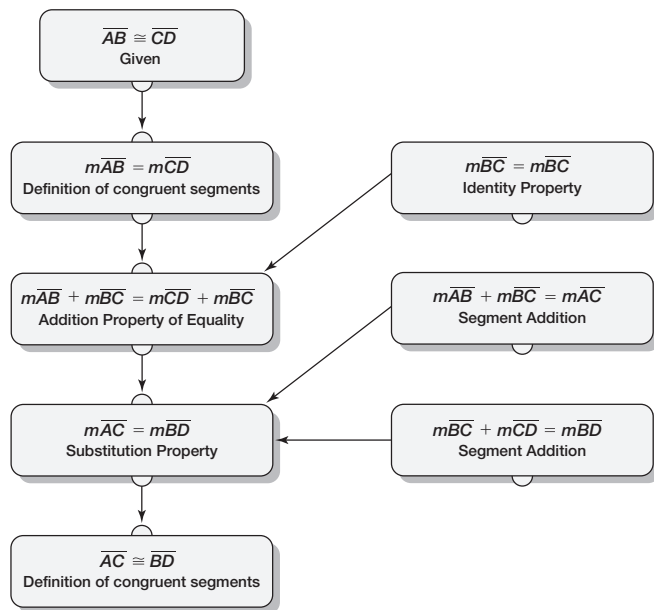Glossary

flow chart proof

A flow chart proof is a proof in which the steps and corresponding reasons are written in boxes. Arrows connect the boxes and indicate how each step and reason is generated from one or more other steps and reasons.

Example

A flow chart proof is shown for the conditional statement: If $\overline{AB} \cong \overline{CD}$, then $\overline{AC} \cong \overline{BD}$.

Given: $\overline{AB} \cong \overline{CD}$

Prove: $\overline{AC} \cong \overline{BD}$

```
                    ┌──────────────────────┐
                    │  AB ≅ CD             │
                    │  Given               │
                    └──────────────────────┘
                              │
                              ▼
  ┌──────────────────────────┐        ┌──────────────────────────┐
  │  mAB = mCD               │        │  mBC = mBC               │
  │  Definition of congruent │        │  Identity Property       │
  │  segments                │        │                          │
  └──────────────────────────┘        └──────────────────────────┘
                │                                    
                ▼                                    
  ┌──────────────────────────────┐    ┌──────────────────────────┐
  │  mAB + mBC = mCD + mBC       │    │  mAB + mBC = mAC         │
  │  Addition Property of Equality│    │  Segment Addition        │
  └──────────────────────────────┘    └──────────────────────────┘
                │                                    
                ▼                                    
  ┌──────────────────────────┐        ┌──────────────────────────┐
  │  mAC = mBD               │◄───────│  mBC + mCD = mBD         │
  │  Substitution Property   │        │  Segment Addition        │
  └──────────────────────────┘        └──────────────────────────┘
                │
                ▼
  ┌──────────────────────────┐
  │  AC ≅ BD                 │
  │  Definition of congruent │
  │  segments                │
  └──────────────────────────┘
```

focus of a parabola

The focus of a parabola is a point such that all points on the parabola are equidistant from the focus and the directrix.

Example

The focus of the parabola shown is the point (0, 2). The directrix of the parabola shown is the line $y = -2$. All points on the parabola are equidistant from the focus and the directrix.
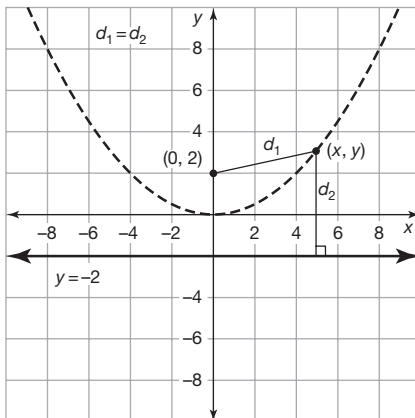
frequency table

A frequency table shows the frequency of an item, number, or event appearing in a sample space.

Example

The frequency table shows the number of times a sum of two number cubes occurred.

Sum of Two Number Cubes	Frequency
2	1
3	2
4	3
5	4
6	5
7	6
8	5
9	4
10	3
11	2
12	1

G

general form of a parabola

The general form of a parabola centered at the origin is an equation of the form $Ax^2 + Dy = 0$ or $By^2 + Cx = 0$.

Example

The equation for the parabola shown can be written in general form as $x^2 - 2y = 0$.
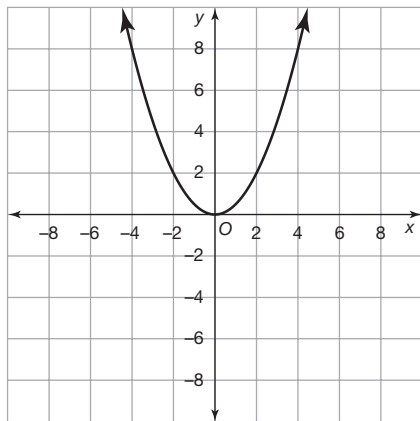
geometric mean

The geometric mean of two positive numbers a and b is the positive number x such that $\frac{a}{x} = \frac{x}{b}$.

Example

The geometric mean of 3 and 12 is 6.

$$\frac{3}{x} = \frac{x}{12}$$
$$x^2 = 36$$
$$x = 6$$

geometric probability

Geometric probability is probability that involves a geometric measure, such as length, area, volume, and so on.

Example

A dartboard has the size and shape shown. The gray shaded area represents a scoring section of the dartboard. Calculate the probability that a dart that lands on a random part of the target will land in a gray scoring section.
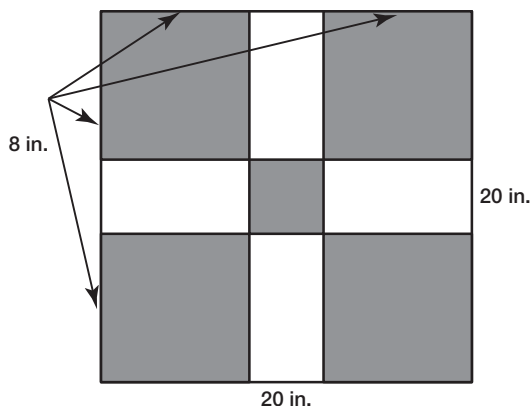
Calculate the area of the dartboard: $20(20) = 400$ in.^{2}

There are 4 gray scoring squares with 8-in. sides and a gray scoring square with $20 - 8 - 8 = 4$-in. sides. Calculate the area of the gray scoring sections: $4(8)(8) + 4(4) = 272$ in.^{2}

Calculate the probability that a dart will hit a gray scoring section: $\frac{272}{400} = 0.68 = 68\%$.

Glossary

great circle of a sphere

The great circle of a sphere is a cross section of a sphere when a plane passes through the center of the sphere.
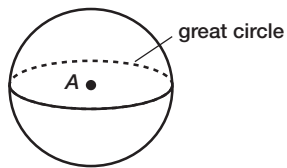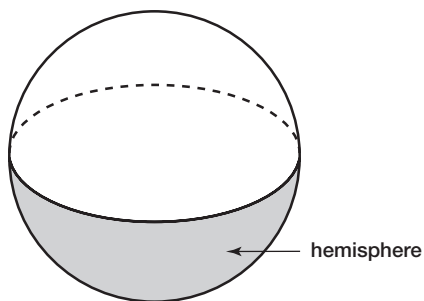
Example

H

hemisphere

A hemisphere is half of a sphere bounded by a great circle.

Example

A hemisphere is shown.

hypothesis

A hypothesis is the "if" part of an "if-then" statement.

Example

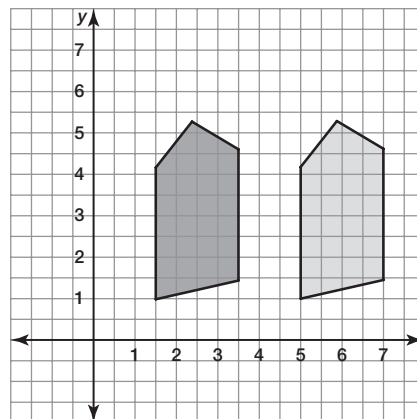In the statement, "If the last digit of a number is a 5, then the number is divisible by 5," the hypothesis is "If the last digit of a number is a 5."

I

image

An image is a new figure formed by a transformation.

Example

The figure on the right is the image that has been created by translating the original figure 3 units to the right horizontally.

incenter

The incenter of a triangle is the point at which the angle bisectors of the triangle intersect.

Example

Point *X* is the incenter of triangle *ABC*.
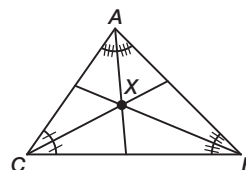
included angle

An included angle is an angle formed by two consecutive sides of a figure.

Example

In triangle *ABC*, angle *A* is the included angle formed by consecutive sides $\overline{AB}$ and $\overline{AC}$.
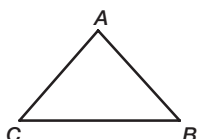
Glossary
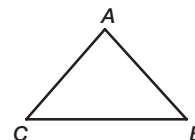
included side

An included side is a line segment between two consecutive angles of a figure.

Example

In triangle ABC, $\overline{AB}$ is the included side formed by consecutive angles A and B.

independent events

Independent events are events for which the occurrence of one event has no impact on the occurrence of the other event.

Example

You randomly choose a yellow marble, replace the marble in the jar, and then randomly choose a yellow marble again. The events of randomly choosing a yellow marble first and randomly choosing a yellow marble second are independent events because the 1st yellow marble was replaced in the jar.

indirect measurement

Indirect measurement is a technique that uses proportions to determine a measurement when direct measurement is not possible.

Example

You can use a proportion to solve for the height x of the flagpole.
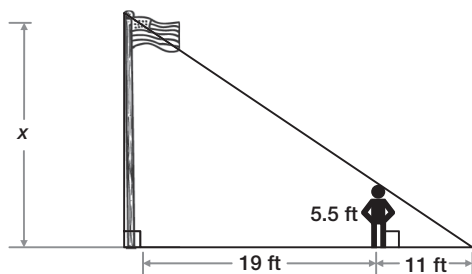
$$\frac{x}{5.5} = \frac{19 + 11}{11}$$
$$\frac{x}{5.5} = \frac{30}{11}$$
$$11x = 165$$
$$x = 15$$

The flagpole is 15 feet tall.

indirect proof or proof by contradiction

An indirect proof, or proof by contradiction, uses the contrapositive. By proving that the contrapositive is true, you prove that the statement is true.

Example

Given: Triangle DEF

Prove: A triangle cannot have more than one obtuse angle.

Given $\triangle DEF$, assume that $\triangle DEF$ has two obtuse angles. So, assume $m\angle D = 91°$ and $m\angle E = 91°$. By the Triangle Sum Theorem, $m\angle D + m\angle E + m\angle F = 180°$. By substitution, $91° + 91° + m\angle F = 180°$, and by subtraction, $m\angle F = -2°$. But, it is not possible for a triangle to have a negative angle, so this is a contradiction. This proves that a triangle cannot have more than one obtuse angle.

induction

induction is reasoning that involves using specific examples to make a conclusion.
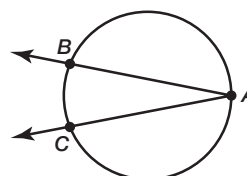
Example

Sandy draws several triangles, measures the interior angles, and calculates the sum of the measures of the three interior angles. She concludes that the sum of the measures of the three interior angles of a triangle is 180°. Sandy reached the conclusion using induction.

inscribed angle

An inscribed angle is an angle whose vertex is on a circle and whose sides contain chords of the circle.

Example

Angle BAC is an inscribed angle. The vertex of angle BAC is on the circle and the sides of angle BAC contain the chords $\overline{AB}$ and $\overline{AC}$.
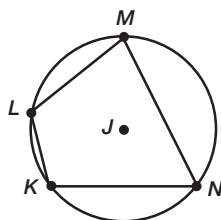
inscribed polygon

An inscribed polygon is a polygon drawn inside a circle such that each vertex of the polygon is on the circle.

Example

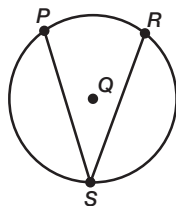Quadrilateral *KLMN* is inscribed in circle *J*.

intercepted arc

An intercepted arc is formed by the intersections of the sides of an inscribed angle with a circle.

Example

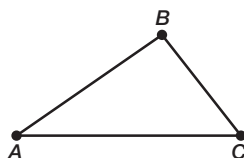$\overline{PR}$ is an intercepted arc of inscribed angle *PSR*.

interior angle of a polygon

An interior angle of a polygon is an angle which is formed by consecutive sides of the polygon or shape.

Example

The interior angles of △*ABC* are ∠*ABC*, ∠*BCA*, and ∠*CAB*.

intersecting sets

Two or more sets are intersecting sets if they have common elements.

Example

Let *V* represent the set of students who are on the girls' volleyball team. Let *M* represent the set of students who are in the math club. Julia is on the volleyball team and belongs to the math club. The sets *V* and *M* are intersecting sets because the two sets have at least one common element, Julia.

inverse

To state the inverse of a conditional statement, negate both the hypothesis and the conclusion.

Conditional Statement: If *p*, then *q*.

Inverse: If not *p*, then not *q*.

Example

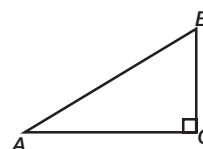Conditional Statement: If a triangle is equilateral, then it is isosceles.

Inverse: If a triangle is not equilateral, then it is not isosceles.

inverse cosine

The inverse cosine, or arc cosine, of *x* is the measure of an acute angle whose cosine is *x*.

Example

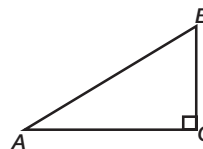In right triangle *ABC*, if cos *A* = *x*, then $\cos^{-1} x = m\angle A$.

inverse sine

The inverse sine, or arc sine, of *x* is the measure of an acute angle whose sine is *x*.

Example

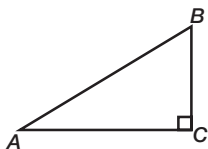In right triangle *ABC*, if sin *A* = *x,* then $\sin^{-1} x = m\angle A$.

inverse tangent

The inverse tangent (or arc tangent) of x is the measure of an acute angle whose tangent is x.

Example

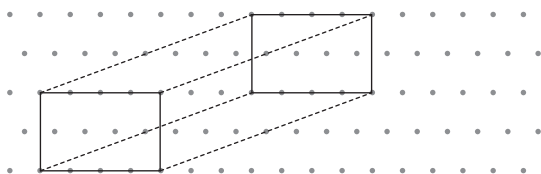In right triangle ABC, if $\tan A = x$, then $\tan^{-1} x = m\angle A$.

isometric paper

Isometric paper is often used by artists and engineers to create three-dimensional views of objects in two dimensions.

Example

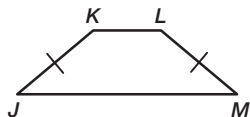The rectangular prism is shown on isometric paper.

isosceles trapezoid

An isosceles trapezoid is a trapezoid whose nonparallel sides are congruent.

Example

In trapezoid $JKLM$, side $\overline{KL}$ is parallel to side $\overline{JM}$, and the length of side $\overline{JK}$ is equal to the length of side $\overline{LM}$, so trapezoid $JKLM$ is an isosceles trapezoid.
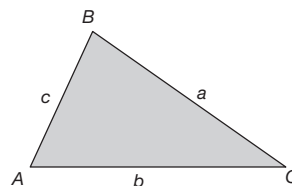
Law of Cosines

The Law of Cosines, or

$$a^2 = c^2 + b^2 - 2bc \cdot \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$

can be used to determine the unknown lengths of sides or the unknown measures of angles in *any* triangle.

Example

In triangle ABC, the measure of angle A is 65°, the length of side b is 4.4301 feet, and the length of side c is 7.6063 feet. Use the Law of Cosines to calculate the length of side a.
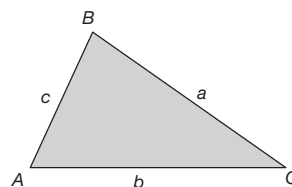
$$a^2 = 4.4301^2 + 7.6063^2 - 2(4.4301)(7.6063) \cos 65°$$

The length of side a is 7 feet.

Law of Sines

The Law of Sines, or $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$, can be used to determine the unknown side lengths or the unknown angle measures in *any* triangle.

Example

In triangle ABC, the measure of angle A is 65°, the measure of angle B is 80°, and the length of side a is 7 feet. Use the Law of Sines to calculate the length of side b.
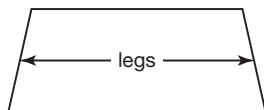
$$\frac{7}{\sin 65°} = \frac{b}{\sin 80°}$$

The length of side b is 7.6063 feet.

legs of a trapezoid

The non-parallel sides of a trapezoid are the legs of the trapezoid.
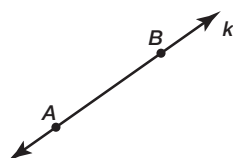
Example

line

A line is made up of an infinite number of points that extend infinitely in two opposite directions. A line is straight and has only one dimension.

Example

The line below can be called line k or line AB.

line segment

A line segment is a portion of a line that includes two points and all of the collinear points between the two points.

Example

The line segment shown is named $\overline{AB}$ or $\overline{BA}$.
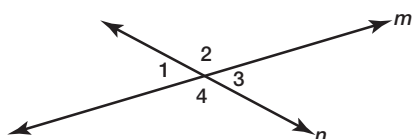

linear pair

A linear pair of angles are two adjacent angles that have noncommon sides that form a line.

Example

The diagram shown has four pairs of angles that form a linear pair.

- Angles 1 and 2 form a linear pair.
- Angles 2 and 3 form a linear pair.
- Angles 3 and 4 form a linear pair.
- Angles 4 and 1 form a linear pair.

linear velocity

Linear velocity is a type of circular velocity described as an amount of distance over a specified amount of time. Linear velocity can be expressed as $v = \frac{s}{t}$, where v = velocity, s = arc length, and t = time.

locus of points

A locus of points is a set of points that satisfy one or more conditions.

Example

A circle is defined as a locus of points that are a fixed distance, called the radius, from a given point, called the center.
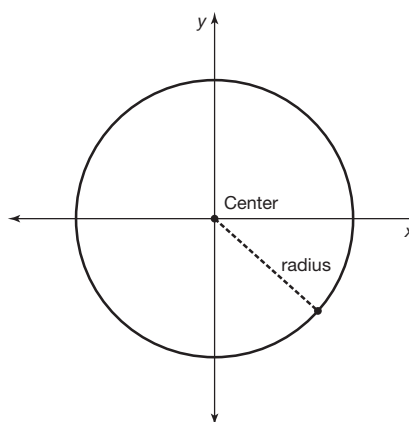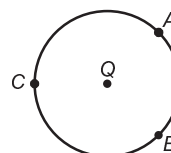

M

major arc

Two points on a circle determine a major arc and a minor arc. The arc with the greater measure is the major arc. The other arc is the minor arc.

Example

Circle Q is divided by points A and B into two arcs, arc ACB and arc AB. Arc ACB has the greater measure, so it is the major arc. Arc AB has the lesser measure, so it is the minor arc.
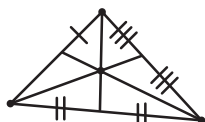
median

The median of a triangle is a line segment drawn from a vertex to the midpoint of the opposite side.

Example

The 3 medians are drawn on the triangle shown.

midpoint

The midpoint of a line segment is the point that divides the line segment into two congruent segments.

Example

Because point B is the midpoint of $\overline{AC}$, $\overline{AB} \cong \overline{BC}$.

Midpoint formula

The Midpoint Formula can be used to calculate the midpoint between two points. The midpoint between (x_1, y_1) and (x_2, y_2) is $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.

Example

To calculate the midpoint between the points $(-1, 4)$ and $(2, -5)$, substitute the coordinates into the Midpoint Formula.

$$\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right) = \left(\dfrac{-1 + 2}{2}, \dfrac{4 - 5}{2}\right)$$
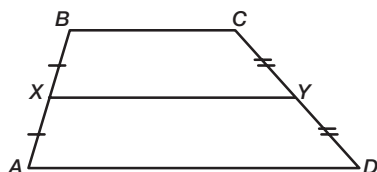
$$= \left(\dfrac{1}{2}, \dfrac{-1}{2}\right)$$

So, the midpoint between the points $(-1, 4)$ and $(2, -5)$ is $\left(\dfrac{1}{2}, -\dfrac{1}{2}\right)$.

midsegment of a trapezoid

The midsegment of a trapezoid is a line segment formed by connecting the midpoints of the legs of the trapezoid.

Example

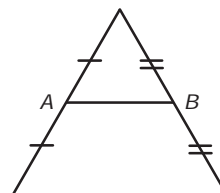Segment XY is the midsegment of trapezoid $ABCD$.

midsegment of a triangle

A midsegment of a triangle is a line segment formed by connecting the midpoints of two sides of a triangle.

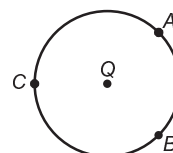**Example**

Segment AB is a midsegment.

minor arc

Two points on a circle determine a minor arc and a major arc. The arc with the lesser measure is the minor arc. The other arc is the major arc.

Example

Circle Q is divided by points A and B into two arcs, arc ACB and arc AB. Arc AB has the lesser measure, so it is the minor arc. Arc ACB has the greater measure, so it is the major arc.


N

non-uniform probability model

When all probabilities in a probability model are not equivalent to each other, it is called a non-uniform probability model.

Example

Spinning the spinner shown represents a non-uniform probability model because the probability of landing on a shaded space is not equal to the probability of landing on a non-shaded space.
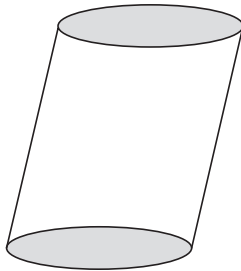
Glossary
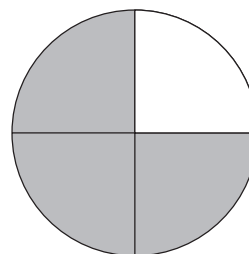
oblique cylinder

When a circle is translated through space in a direction that is not perpendicular to the plane containing the circle, the solid formed is an oblique cylinder.
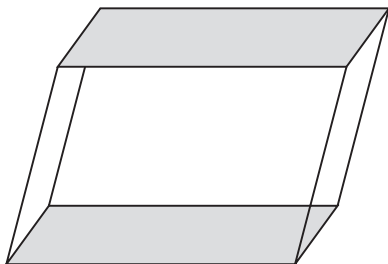
Example

The prism shown is an oblique cylinder.

oblique rectangular prism

When a rectangle is translated through space in a direction that is not perpendicular to the plane containing the rectangle, the solid formed is an oblique rectangular prism.
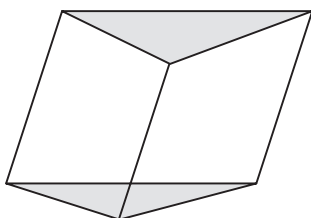
Example

The prism shown is an oblique rectangular prism.
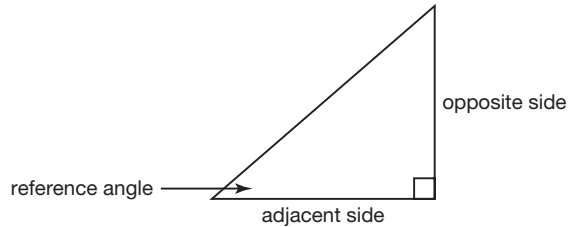
oblique triangular prism

When a triangle is translated through space in a direction that is not perpendicular to the plane containing the triangle, the solid formed is an oblique triangular prism.

Example

The prism shown is an oblique triangular prism.

opposite side

The opposite side of a triangle is the side opposite the reference angle.

Example

organized list

An organized list is a visual model for determining the sample space of events.

Example

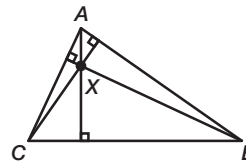The sample space for flipping a coin 3 times can be represented as an organized list.

HHH	THH
HHT	THT
HTH	TTH
HTT	TTT

orthocenter

The orthocenter of a triangle is the point at which the altitudes of the triangle intersect.

Example

Point X is the orthocenter of triangle ABC.

outcome

An outcome is the result of a single trial of an experiment.

Example

Flipping a coin has two outcomes: heads or tails.

parabola

A parabola is the set of all points in a plane that are equidistant from a fixed point called the focus and a fixed line called the directrix.

Example

The focus of the parabola shown is the point (0, 2). The directrix of the parabola shown is the line $y = -2$. All points on the parabola are equidistant from the focus and the directrix.
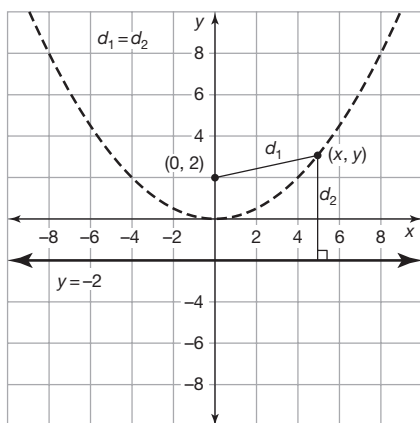

paragraph proof

A paragraph proof is a proof that is written in paragraph form. Each sentence includes mathematical statements that are organized in logical steps with reasons.

Example

The proof shown is a paragraph proof that vertical angles 1 and 3 are congruent.

Angle 1 and angle 3 are vertical angles. By the definition of linear pair, angle 1 and angle 2 form a linear pair. Angle 2 and angle 3 also form a linear pair. By the Linear Pair Postulate, angle 1 and angle 2 are supplementary. Angle 2 and angle 3 are also supplementary. Angle 1 is congruent to angle 3 by the Congruent Supplements Theorem.
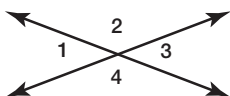
permutation

A permutation is an ordered arrangement of items without repetition.

Example

The permutations of the letters *A*, *B*, and *C* are:

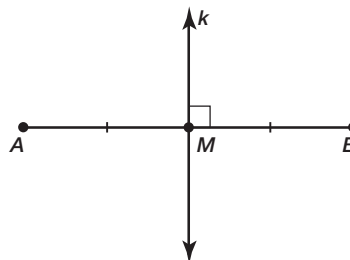|	
ABC	*ACB*
BAC	*BCA*
CAB	*CBA*

perpendicular bisector

A perpendicular bisector is a line, line segment, or ray that intersects the midpoint of a line segment at a 90-degree angle.

Example

Line *k* is the perpendicular bisector of $\overline{AB}$. It is perpendicular to $\overline{AB}$, and intersects $\overline{AB}$ at midpoint *M* so that $AM = MB$.

plane

A plane is a flat surface with infinite length and width, but no depth. A plane extends infinitely in all directions.

Example

Plane *A* is shown.

point

A point has no dimension, but can be visualized as a specific position in space, and is usually represented by a small dot.

Example
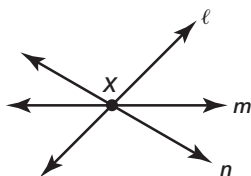
point *A* is shown.

Glossary

point of concurrency

A point of concurrency is the point at which three or more lines intersect.
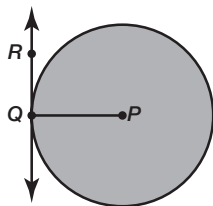
Example

Point X is the point of concurrency for lines ℓ, m, and n.

point of tangency

A tangent to a circle is a line that intersects the circle at exactly one point, called the point of tangency.

Example

Line RQ is tangent to circle P. Point Q is the point of tangency.

point-slope form

The point-slope form of a linear equation that passes through the point (x_1, y_1) and has slope m is $y - y_1 = m(x - x_1)$.

Example

A line passing through the point $(1, 2)$ with a slope of $\frac{1}{2}$ can be written in point-slope form as $y - 2 = \frac{1}{2}(x + 1)$.

postulate

A postulate is a statement that is accepted to be true without proof.
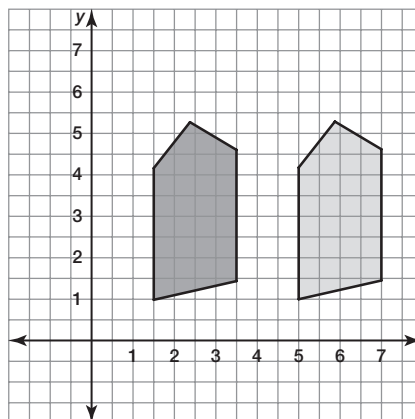
Example

The following statement is a postulate: A straight line may be drawn between any two points.

pre-image

A pre-image is the figure that is being transformed.

Example

The figure on the right is the image that has been formed by translating the pre-image 3 units to the right horizontally.

probability

The probability of an event is the ratio of the number of desired outcomes to the total number of possible outcomes, $P(A) = \frac{\text{desired outcomes}}{\text{possible outcomes}}$.

Example

When flipping a coin, there are 2 possible outcomes: heads or tails. The probability of flipping a heads is $\frac{1}{2}$.

probability model

A probability model lists the possible outcomes and the probability for each outcome. In a probability model, the sum of the probabilities must equal 1.

Example

The table shows a probability model for flipping a fair coin once.

Outcomes	Heads (H)	Tails (T)
Probability	$\frac{1}{2}$	$\frac{1}{2}$

propositional form

When a conditional statement is written using the propositional variables p and q, the statement is said to be written in propositional form.

Example

Propositional form:
"If p, then q."
$p \rightarrow q$

propositional variables

When a conditional statement is written in propositional form as "If p, then q," the variables p and q are called propositional variables.
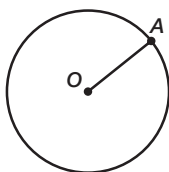
R

radian

One radian is defined as the measure of a central angle whose arc length is the same as the radius of the circle.

radius

The radius of a circle is a line segment with one endpoint on the circle and one endpoint at the center.
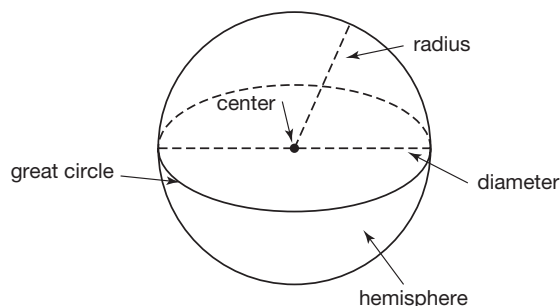
Example

In circle O, $\overline{OA}$ is a radius.

radius of a sphere

The radius of a sphere is a line segment with one endpoint on the sphere and one endpoint at the center.

Example

rationalizing the denominator

Rationalizing the denominator is the process of eliminating a radical from the denominator of an expression. To rationalize the denominator, multiply by a form of one so that the radicand of the radical in the denominator is a perfect square.

Example

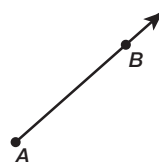Rationalize the denominator of the expression $\frac{5}{\sqrt{3}}$.

$$\frac{5}{\sqrt{3}} = \frac{5}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$
$$= \frac{5\sqrt{3}}{\sqrt{9}}$$
$$= \frac{5\sqrt{3}}{3}$$

ray

A ray is a portion of a line that begins with a single point and extends infinitely in one direction.
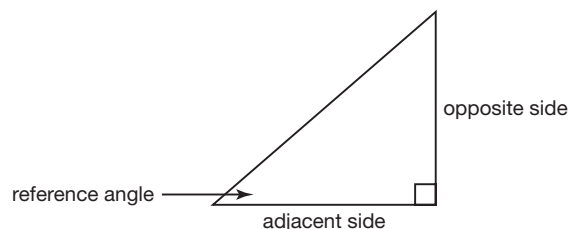
Example

The ray shown is ray AB.

reference angle

A reference angle is the angle of the right triangle being considered. The opposite side and adjacent side are named based on the reference angle.

Example

Reflexive Property

The reflexive property states that $a = a$.

Example

The statement $2 = 2$ is an example of the reflexive property.

Glossary

relative frequency

A relative frequency is the ratio or percent of occurrences within a category to the total of the category.
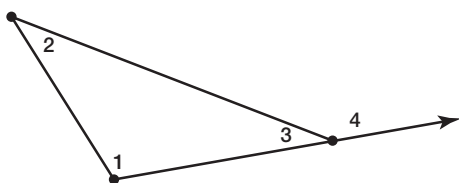
Example

John surveys 100 students in his school about their favorite school subject. Of the 100 students, 37 chose math as their favorite subject. The relative frequency of students show selected math as their favorite subject is $\frac{37}{100}$, or 37%.

remote interior angles of a triangle

The remote interior angles of a triangle are the two angles that are not adjacent to the specified exterior angles.
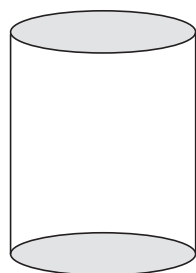
Example

The remote interior angles with respect to exterior angles 4 are angles 1 and 2.

right cylinder

A disc translated through space in a direction perpendicular to the plane containing the disc forms a right cylinder.
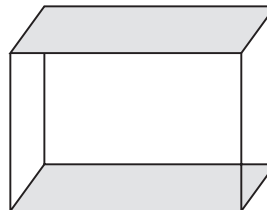
Example

right rectangular prism

A rectangle translated through space in a direction perpendicular to the plane containing the rectangle forms a right rectangular prism.
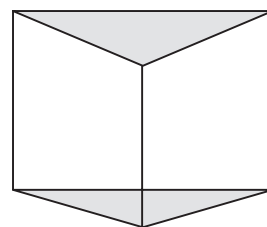
Example

right triangular prism

A triangle translated through space in a direction perpendicular to the plane containing the triangle forms a right triangular prism.

Example

rigid motion

A rigid motion is a transformation of points in space. Translations, reflections, and rotations are examples of rigid motion.

Rule of Compound Probability involving "and"

The Rule of Compound Probability involving "and" states: "If Event A and Event B are independent, then the probability that Event A happens *and* Event B happens is the product of the probability that Event A happens and the probability that Event B happens, given that Event A has happened."

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Example

You flip a coin two times. Calculate the probability of flipping a heads on the first flip and flipping a heads on the second flip.

Let A represent the event of flipping a heads on the first flip. Let B represent the event of flipping a heads on the second flip.

$P(A \text{ and } B) = P(A) \cdot P(B)$

$P(A \text{ and } B) = \frac{1}{2} \cdot \frac{1}{2}$

$P(A \text{ or } B) = \frac{1}{4}$

So, the probability of flipping a heads on the first flip and flipping a heads on the second flip is $\frac{1}{4}$.

S

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sample space

A list of all possible outcomes of an experiment is called a sample space.

Example

Flipping a coin two times consists of four outcomes: HH, HT, TH, and TT.
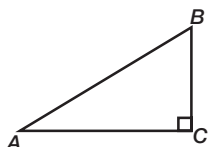
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secant (sec)

The secant (sec) of an acute angle in a right triangle is the ratio of the length of the hypotenuse to the length of the side adjacent to the angle.

Example

In triangle ABC, the secant of angle A is:

$$\sec A = \frac{\text{length of hypotenuse}}{\text{length of side adjacent to } \angle A} = \frac{AB}{AC}$$

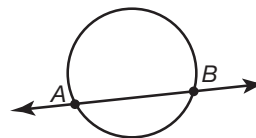The expression "sec A" means "the secant of angle A."

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secant of a circle

A secant of a circle is a line that intersects the circle at two points.

Example

The line intersecting the circle through points A and B is a secant.
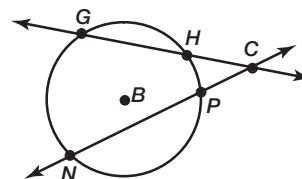

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secant segment

A secant segment is formed when two secants intersect outside of a circle. A secant segment begins at the point at which the two secants intersect, continues into the circle, and ends at the point at which the secant exits the circle.

Example

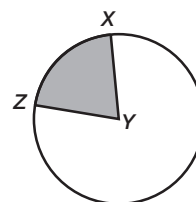Segment GC and segment NC are secant segments.

---

sector of a circle

A sector of a circle is a region of the circle bounded by two radii and the included arc.

Example

In circle Y, arc XZ, radius XY, and radius YZ form a sector.
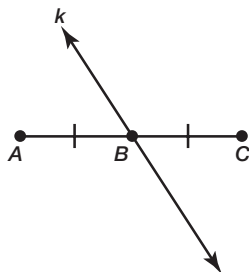
Glossary

segment bisector

A segment bisector is a line, line segment, or ray that intersects a line segment so that the line segment is divided into two segments of equal length.

Example

Line *k* is a segment bisector of segment *AC*. The lengths of segments *AB* and *BC* are equal.
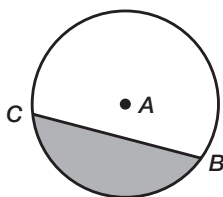
segment of a circle

A segment of a circle is a region bounded by a chord and the included arc.

Example

In circle *A,* chord $\overline{BC}$ and arc *BC* are the boundaries of a segment of the circle.
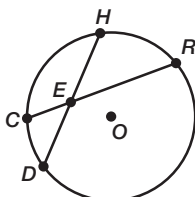
segments of a chord

Segments of a chord are the segments formed on a chord if two chords of a circle intersect.

Example

The segments of chord $\overline{HD}$ are $\overline{EH}$ and $\overline{ED}$.
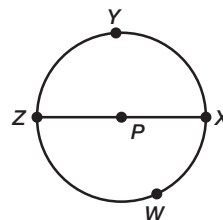The segments of chord $\overline{RC}$ are $\overline{ER}$ and $\overline{EC}$.

semicircle

A semicircle is an arc whose endpoints form the endpoints of a diameter of the circle.

Example

Arc *XYZ* and arc *ZWX* are semicircles of circle *P*.

set

A set is a collection of items. If *x* is a member of set *B*, then *x* is an element of set *B*.

Example

Let *E* represent the set of even whole numbers.
$E = \{2, 4, 6, 8, . . .\}$

similar triangles

Similar triangles are triangles that have all pairs of corresponding angles congruent and all corresponding sides are proportional.

Example

Triangle *ABC* is similar to triangle *DEF*.
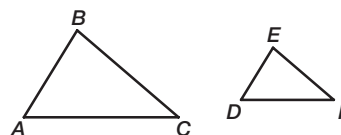
simulation

A simulation is an experiment that models a real-life situation.

Example

You can simulate the selection of raffle numbers by using the random number generator on a graphing calculator.

sine (sin)

The sine (sin) of an acute angle in a right triangle is the ratio of the length of the side opposite the angle to the length of the hypotenuse.

Example

In triangle ABC, the sine of angle A is:

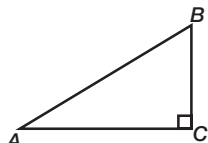$$\sin A = \frac{\text{length of side opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AB}$$

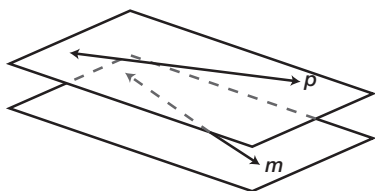The expression "sin A" means "the sine of angle A."

sketch

To sketch is to create a geometric figure without using tools such as a ruler, straightedge, compass, or protractor. A drawing is more accurate than a sketch.

skew lines

Skew lines are two lines that do not intersect and are not parallel. Skew lines do not lie in the same plane.

Example

Line m and line p are skew lines.

sphere

A sphere is the set of all points in space that are a given distance from a fixed point called the center of the sphere.

Example

A sphere is shown.
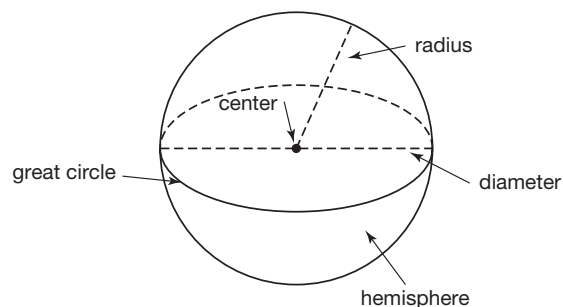
standard form of a parabola

The standard form of a parabola centered at the origin is an equation of the form $x^2 = 4py$ or $y^2 = 4px$, where p represents the distance from the vertex to the focus.

Example

The equation for the parabola shown can be written in standard form as $x^2 = 2y$.
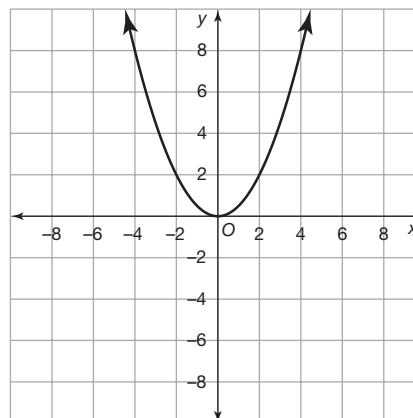
straightedge

A straightedge is a ruler with no numbers.

Substitution Property of Equality

The Substitution Property of Equality states: "If a and b are real numbers and $a = b$, then a can be substituted for b."

Example

If $AB = 12$ ft and $CD = 12$ ft, then $AB = CD$.

Glossary

Subtraction Property of Equality

The Subtraction Property of Equality states: "If $a = b$, then $a - c = b - c$."

Example

If $x + 5 = 7$, then $x + 5 - 5 = 7 - 5$, or $x = 2$ is an example of the subtraction property of equality.

supplementary angles

Two angles are supplementary if the sum of their measures is 180°.

Example

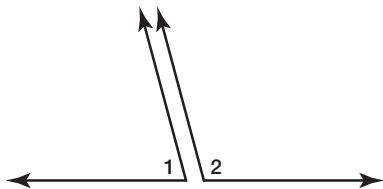Angle 1 and angle 2 are supplementary angles.

If $m\angle 1 = 75°$, then $m\angle 2 = 180° - 75° = 105°$.
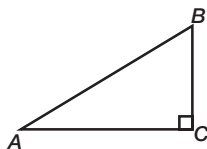

T

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tangent (tan)

The tangent (tan) of an acute angle in a right triangle is the ratio of the length of the side opposite the angle to the length of the side adjacent to the angle.

Example

In triangle ABC, the tangent of angle A is:

$$\tan A = \frac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A} = \frac{BC}{AC}$$

The expression "tan A" means "the tangent of angle A."

tangent of a circle

A tangent of a circle is a line that intersects the circle at exactly one point, called the point of tangency.

Example

Line RQ is tangent to circle P.
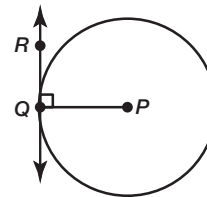
tangent segment

A tangent segment is a line segment formed by connecting a point outside of the circle to a point of tangency.

Example

Line segment AB and line segment AC are tangent segments.
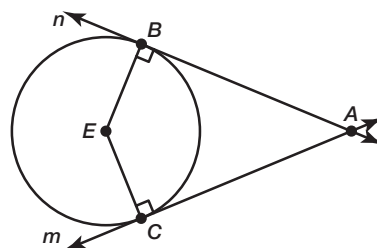
theorem

A theorem is a statement that has been proven to be true.

Example

The Pythagorean Theorem states that if a right triangle has legs of lengths a and b and hypotenuse of length c, then $a^2 + b^2 = c^2$.

theoretical probability

Theoretical probability is the mathematical calculation that an event will happen in theory.

Example

The theoretical probability of rolling a 1 on a number cube is $\frac{1}{6}$.

Glossary

transformation

A transformation is an operation that maps, or moves, a figure, called the preimage, to form a new figure called the image. Three types of transformations are reflections, rotations, and translations.

Example

reflection over a line

rotation about a point

translation

Transitive Property of Equality

The Transitive Property of Equality states: "If $a = b$ and $b = c$, then $a = c$."

Example

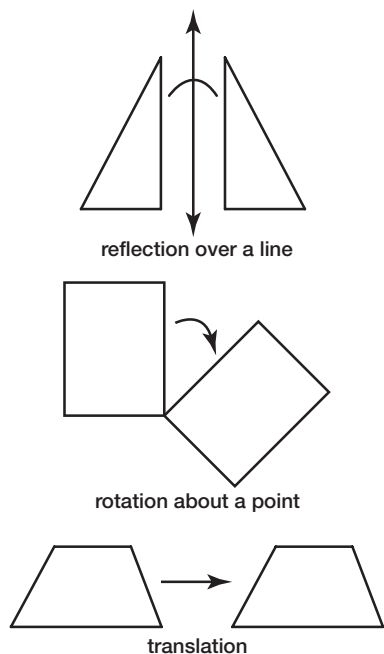If $x = y$ and $y = 2$, then $x = 2$ is an example of the Transitive Property of Equality.

translation

A translation is a transformation in which a figure is shifted so that each point of the figure moves the same distance in the same direction. The shift can be in a horizontal direction, a vertical direction, or both.

Example

The top trapezoid is a vertical translation of the bottom trapezoid by 5 units.
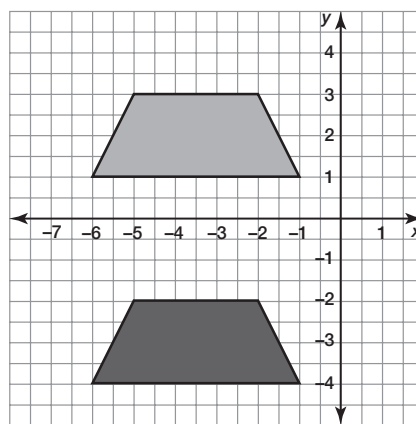
tree diagram

A tree diagram is a diagram that illustrates sequentially the possible outcomes of a given situation.

Example

Glossary

truth table

A truth table is a table that summarizes all possible truth values for a conditional statement $p \rightarrow q$. The first two columns of a truth table represent all possible truth values for the propositional variables p and q. The last column represents the truth value of the conditional statement $p \rightarrow q$.

Example

The truth value of the conditional statement $p \rightarrow q$ is determined by the truth value of p and the truth value of q.

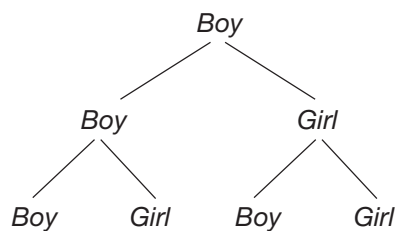- If p is true and q is true, then $p \rightarrow q$ is true.
- If p is true and q is false, then $p \rightarrow q$ is false.
- If p is false and q is true, then $p \rightarrow q$ is true.
- If p is false and q is false, then $p \rightarrow q$ is true.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

truth value

The truth value of a conditional statement is whether the statement is true or false. If a conditional statement could be true, then the truth value of the statement is considered true. The truth value of a conditional statement is either true or false, but not both.

Example

The truth value of the conditional statement "If a quadrilateral is a rectangle, then it is a square" is false.

two-column proof

A two-column proof is a proof consisting of two columns. In the left column are mathematical statements that are organized in logical steps. In the right column are the reasons for each mathematical statement.

Example

The proof shown is a two-column proof.

Statements	Reasons
1. ∠1 and ∠3 are vertical angles.	**1.** Given
2. ∠1 and ∠2 form a linear pair. ∠2 and ∠3 form a linear pair.	**2.** Definition of linear pair
3. ∠1 and ∠2 are supplementary. ∠2 and ∠3 are supplementary.	**3.** Linear Pair Postulate
4. ∠1 ≅ ∠3	**4.** Congruent Supplements Theorem

two-way frequency table

A two-way frequency table, also called a contingency table, shows the number of data points and their frequencies for two variables. One variable is divided into rows, and the other is divided into columns.

Example

The two-way frequency table shows the hand(s) favored by people who do and do not participate in individual or team sports.

Sports Participation

Favored Hand		Individual	Team	Does Not Play	Total
	Left	3	13	8	24
	Right	6	23	4	33
	Mixed	1	3	2	6
	Total	10	39	14	63

Glossary

two-way relative frequency table

A two-way relative frequency table displays the relative frequencies for two categories of data.

Example

The two-way relative frequency table shows the hand(s) favored by people who do and do not participate in individual or team sports.

	Individual	Team	Does Not Play	Total
Left	$\frac{3}{63} \approx 4.8\%$	$\frac{13}{63} \approx 20.6\%$	$\frac{8}{63} \approx 12.7\%$	$\frac{24}{63} \approx 38.1\%$
Right	$\frac{6}{63} \approx 9.5\%$	$\frac{23}{63} \approx 36.5\%$	$\frac{4}{63} \approx 6.3\%$	$\frac{33}{63} \approx 52.4\%$
Mixed	$\frac{1}{63} \approx 1.6\%$	$\frac{3}{63} \approx 4.8\%$	$\frac{2}{63} \approx 3.2\%$	$\frac{6}{63} \approx 9.5\%$
Total	$\frac{10}{63} \approx 15.9\%$	$\frac{39}{63} \approx 61.9\%$	$\frac{14}{63} \approx 22.2\%$	$\frac{63}{63} = 100\%$

two-way table

A two-way table shows the relationship between two data sets, one data set is organized in rows and the other data set is organized in columns.

Example

The two-way table shows all the possible sums that result from rolling two number cubes once.

2nd Number Cube

1st Number Cube		1	2	3	4	5	6
	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

Glossary

U

uniform probability model

A uniform probability model occurs when all the probabilities in a probability model are equally likely to occur.
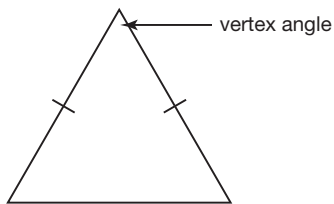
Example

Rolling a number cube represents a uniform probability model because the probability of rolling each number is equal.

V

vertex angle of an isosceles triangle

The vertex angle of an isosceles triangle is the angle formed by the two congruent legs.
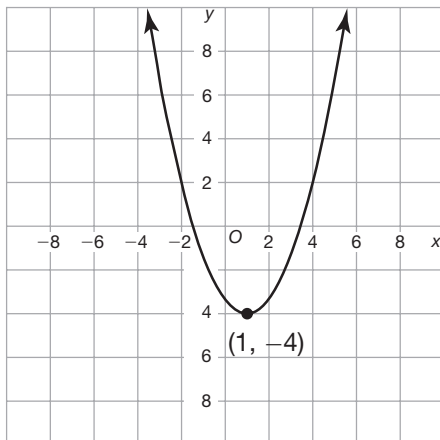
Example

vertex angle

vertex of a parabola

The vertex of a parabola, which lies on the axis of symmetry, is the highest or lowest point on the parabola.

Example

The vertex of the parabola is the point $(1, -4)$, the minimum point on the parabola.

$(1, -4)$

vertical angles

Vertical angles are two nonadjacent angles that are formed by two intersecting lines.

Examples

Angles 1 and 3 are vertical angles.
Angles 2 and 4 are vertical angles.