Coach C's Notes

On a Jet Plane

3.4

Area and Perimeter of Trapezoids on the Coordinate Plane

LEARNING GOALS

In this lesson, you will:

- Determine the perimeter and the area of trapezoids and hexagons on a coordinate plane.
- Determine the perimeter of composite figures on the coordinate plane.
- Determine and describe how proportional changes in the linear dimensions of a trapezoid affect its perimeter and area.

KEY TERMS

- bases of a trapezoid
- legs of a trapezoid

How can you make a building withstand an earthquake? The ancient Incas figured out a way—by making trapezoidal doors and windows.

The Inca Empire expanded along the South American coast—an area that experiences a lot of earthquakes—from the 12th through the late 15th century.

One of the most famous of Inca ruins is Machu Picchu in Peru. There you can see the trapezoidal doors and windows—tilting inward from top to bottom to better withstand the seismic activity.

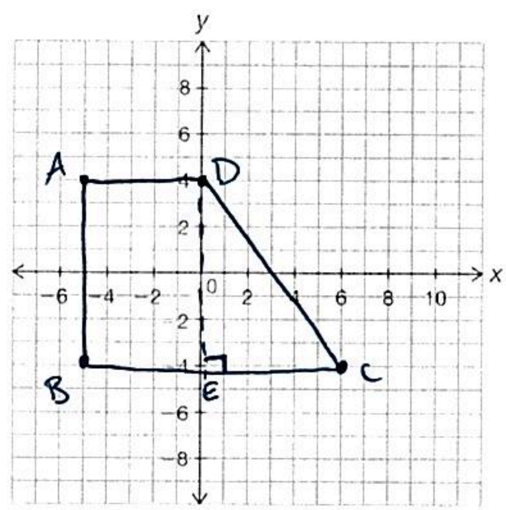
PROBLEM 1 Well, It's the Same, But It's Also Different!

So far, you have determined the perimeter and the area of parallelograms—including rectangles and squares. Now, you will move on to trapezoids.

1. Plot each point in the coordinate plane shown:

- $A(-5, 4)$
- $B(-5, -4)$
- $C(6, -4)$
- $D(0, 4)$

Then, connect the points in alphabetical order.



2. Explain how you know that the quadrilateral you graphed is a trapezoid.

one pair of parallel sides

The trapezoid is unique in the quadrilateral family because it is a quadrilateral that has *exactly* one pair of parallel sides. The parallel sides are known as the **bases of the trapezoid**, while the non-parallel sides are called the **legs of the trapezoid**.

3. Using the trapezoid you graphed, identify:

   a. the bases.

   $\overline{AD}$ and $\overline{BC}$

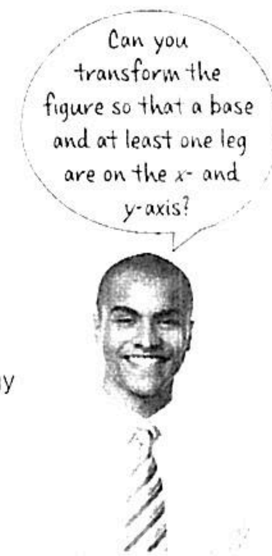
   b. the legs.

   $\overline{AB}$ and $\overline{DC}$

4. Analyze trapezoid *ABCD* that you graphed on the coordinate plane.

   a. Describe how you can determine the perimeter of trapezoid *ABCD* without using the Distance Formula.

   Find the 3 side lengths that have vertical and horizontal distances. Find the last leg using Pythagorean theorem.



   b. Determine the perimeter of trapezoid *ABCD* using the strategy you described in part (a). First, perform a transformation of trapezoid *ABCD* on the coordinate plane and then calculate the perimeter of the image.

$AB = 4 - (-4) = 8$

$BC = 6 - (-5) = 11$

$AD = 0 - (-5) = 5$

$a^2 + b^2 = c^2$

$AB^2 + CE^2 = DC^2$

$8^2 + 6^2 = DC^2$

$64 + 36 = DC^2$

$100 = DC^2$

$\sqrt{100} = DC$

$DC = 10$

$P = AB + BC + AD + DC$

$P = 8 + 11 + 5 + 10$

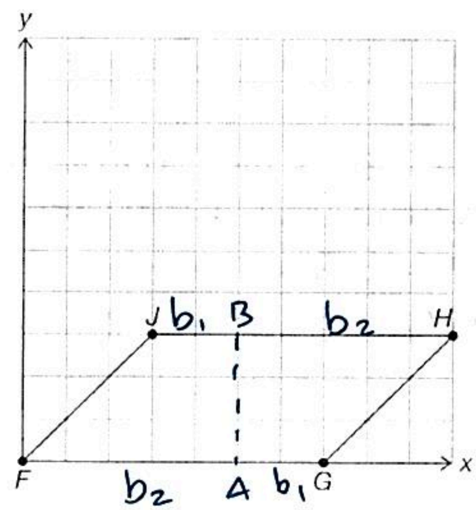
$\boxed{P = 34}$

PROBLEM 2 Using What You Know

So, what similarities are there between determining the area of a parallelogram and determining the area of a trapezoid?

Recall that the formula for the area of a parallelogram is $A = bh$, where b represents the base and h represents the height. As you know, a parallelogram has both pairs of opposite sides parallel. But what happens if you divide the parallelogram into two congruent trapezoids?

1. Analyze parallelogram $FGHJ$ on the coordinate plane.



a. Divide parallelogram $FGHJ$ into two congruent trapezoids.

b. Label the two vertices that make up the two congruent trapezoids.

c. Label the bases that are congruent to each other. Label one pair of bases b_1 and the other pair b_2.

d. Now write a formula for the area of the entire geometric figure. Make sure you use the bases you labeled and do not forget the height.

$A = bh$

Substitute $b_1 + b_2$ for b = $A = (b_1 + b_2)h$

e. Now write the formula for the area for *half* of the entire figure.

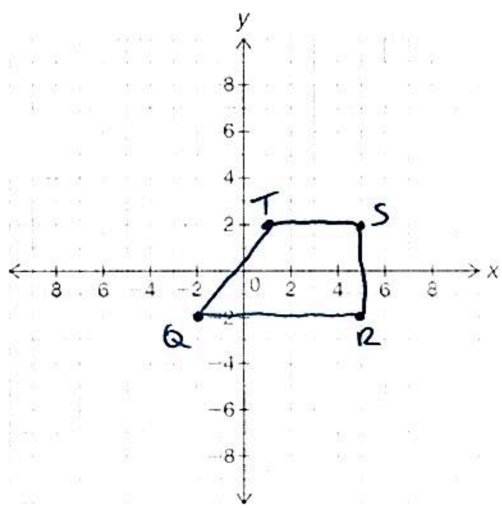
$A = \left(\frac{b_1 + b_2}{2}\right)h \leftarrow$ Area of a Trapezoid

2. What can you conclude about the area formula of a parallelogram and the area formula of a trapezoid? Why do you think this connection exists?

3. Plot each point on the coordinate plane shown:

- $Q(-2, -2)$
- $R(5, -2)$
- $S(5, 2)$
- $T(1, 2)$

Then, connect the points in alphabetical order.



4. Determine the area of trapezoid $QRST$. Describe the strategy or strategies you used to determine your answer.

$b_1 = QR = 5-(-2) = 7$

$b_2 = TS = 5-1 = 4$

$h = SR = 2-(-2) = 4$

$A = \left(\frac{b_1 + b_2}{2}\right)h$

$A = \left(\frac{4+7}{2}\right)4$

$A = \left(\frac{11}{2}\right)4$

$\boxed{A = 22}$

PROBLEM 4 Reaching for New Heights, and Bases . . . One Last Time

Recall that you have determined and described how proportional changes in the linear dimensions of rectangles, triangles, and parallelograms affect their perimeter and area. You can apply that knowledge to trapezoids.

1. Describe how multiplying the bases and height of a trapezoid by a factor of $\frac{1}{4}$ will affect the area of the resulting trapezoid. Provide an example, determine its area, and explain your reasoning.

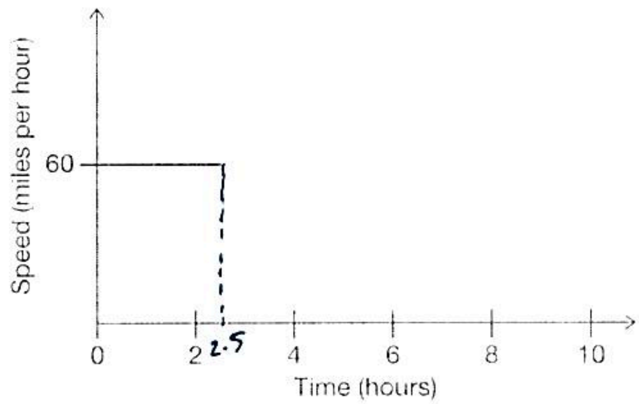
dimensions		Area
$\frac{1}{4}$	$\longrightarrow$	$\frac{1}{16}$

Areas changes by the factor sqrared

3

Jets and Trapezoids!

The graph shows the constant speed of a car on the highway over the course of 2.5 hours.

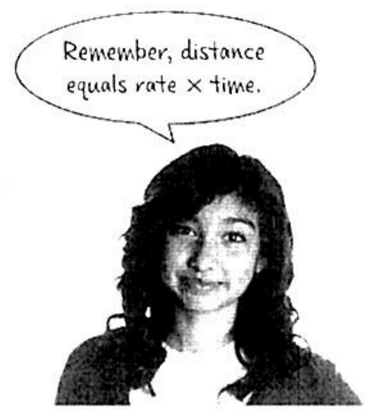


1. Describe how you could calculate the distance the car traveled in 2.5 hours using what you know about area.

   Vertical line to create rectangle

   distance = rate × time

   Area = base × height

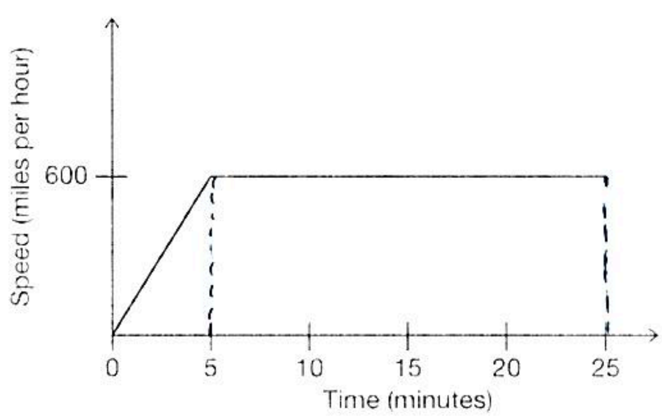


2. How far did the car travel in 2.5 hours?

   $2.5 \times 60 = 150$ miles

The graph you used is called a velocity-time graph. In a velocity-time graph, the area under the line or curve gives the distance.

The graph shown describes the speed and the time of a passenger jet's ascent.



3. How can you use the graph to determine the distance the jet has traveled in 25 minutes?

vertical line to 25 min.

4. What shape is the region on the graph enclosed by the line segments?

trapezoid

5. Determine the distance the jet has traveled in 25 minutes. Show your work.

$$A = \left(\frac{b_1 + b_2}{2}\right)h$$

$$A = \frac{1}{2}(b_1 + b_2)h$$

$$A = \frac{1}{2}\left(\frac{25}{60} + \frac{20}{60}\right)600$$

$$A = \frac{1}{2}\left(\frac{45}{60}\right)600$$

$$A = \frac{1}{2}(45)(10) \quad \boxed{A = 225 \text{ miles in } 25 \text{ min}}$$

6. Determine the distance the jet has traveled in 5 minutes. Show your work.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}\left(\frac{5}{60}\right)(600)$$

$$A = \frac{1}{2}(5)(10)$$

$$A = 25$$

Be prepared to share your solutions and methods.