Coach C's Notes

9.1 - 9.4 Notes
Intro to Trigonometry, Trigonometric Ratios, and Inverse Trigonometry

Trigonometry Vocab:

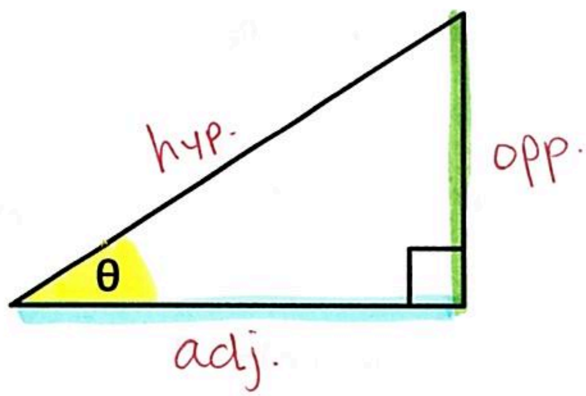
A trigonometric ratio compares the lengths of two sides of a right triangle with a reference angle.

The reference angle is the angle of the triangle you are looking at or using in a problem. The Greek letter theta "θ" is traditionally used to represent the measure of the reference angle, an acute angle in a right triangle. The values of trigonometric ratios depend on θ.

You know that the hypotenuse of a right triangle is the side that is opposite the right angle. In trigonometry, the legs of a right triangle are often referred to as the *opposite side* and the *adjacent side*.

The opposite side is the side opposite the *reference angle.*

The adjacent side is the side adjacent to the *reference angle* that is not the hypotenuse.



Ratios:

Trigonometric Functions

	WORDS
SOH	The **sine** (sin) of angle θ is the ratio of the length of the opposite leg to the length of the hypotenuse.
CAH	The **cosine** (cos) of angle θ is the ratio of the length of the adjacent leg to the length of the hypotenuse.
TOA	The **tangent** (tan) of angle θ is the ratio of the length of the opposite leg to the length of the adjacent leg.

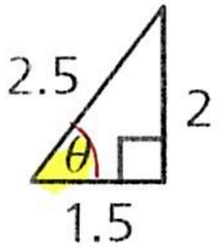
$$\sin(\theta) = \frac{\text{opp.}}{\text{hyp.}}$$

$$\cos(\theta) = \frac{\text{adj.}}{\text{hyp.}}$$

$$\tan(\theta) = \frac{\text{opp.}}{\text{adj.}}$$

SOH CAH TOA

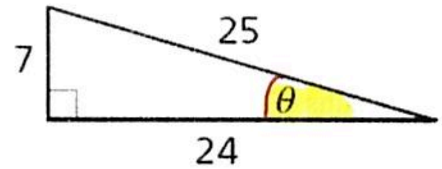
Write the ratio for the three trig functions for the triangles below:



$\sin\theta = \frac{2}{2.5}$

$\cos\theta = \frac{1.5}{2.5}$

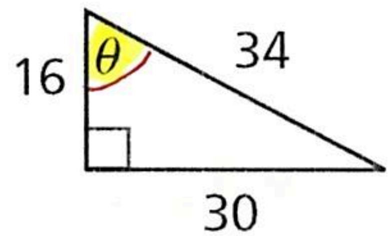
$\tan\theta = \frac{2}{1.5}$



$\sin\theta = \frac{7}{25}$

$\cos\theta = \frac{24}{25}$

$\tan\theta = \frac{7}{24}$



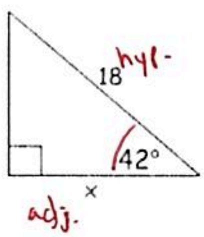
$\sin\theta = \frac{30}{34}$

$\cos\theta = \frac{16}{34}$

$\tan\theta = \frac{30}{16}$

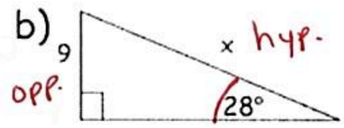
For the triangles below decide which trig function you would use based on the information given and write a ratio for each: SOH CAH TOA

a)



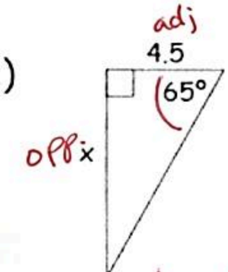
$\cos(42) = \frac{x}{18}$

b)



$\sin(28) = \frac{9}{x}$

c)



$\tan(65) = \frac{x}{4.5}$

The values of sine, cosine and tangent depend only on angle θ and not on the size of the triangle. No matter which triangle is used, the value of $\sin\theta$ will be the same as long as the measure of the reference angle, θ, is the same. The same applies for all trigonometric functions.

For example, for any right triangle with an angle of 40°, if you take the measure of the **opposite** side and divide by the length of the **hypotenuse** you will get $\approx .64$ which is the **sine** of 40°.

For any right triangle with an angle of 40°, if you take the measure of the **adjacent** side and divide by the length of the **hypotenuse** you will get $\approx .77$ which is the **cosine** of 40°.

For any right triangle with an angle of 40°, if you take the measure of the **opposite** side and divide by the measure of the **adjacent** side you will get $\approx .84$ which is the **tangent** of 40°.

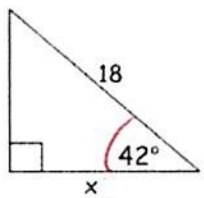
Caution!

Make sure that your graphing calculator is set to interpret angle values as degrees. Press MODE. Check that **Degree** and not **Radian** is highlighted in the third row.

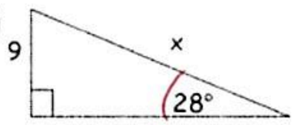
Solving Trig Functions:

Using a calculator and trigonometric ratios, solve for x:

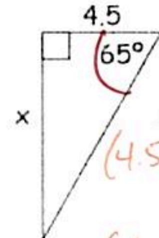
a)



b)



c)



$(18)\cos(42) = \frac{x}{\cancel{18}}$

$(18)\cos(42) = x$

$\boxed{x = 13.38}$

$(x)\sin(28) = \frac{9}{\cancel{x}}(x)$

$\frac{(x)\cancel{\sin(28)}}{\cancel{\sin(28)}} = \frac{9}{\sin(28)}$

$x = \frac{9}{\sin(28)}$ $\boxed{x = 19.17}$

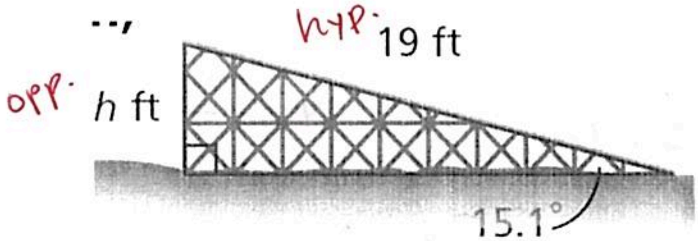
$(4.5)\tan(65) = \frac{x}{\cancel{4.5}}$

$(4.5)\tan(65) = x$

$\boxed{x = 9.65}$

Word Problems:

Example 1: In a waterskiing competition, a jump ramp is 19ft long and has a 15.1° angle with the water. To the nearest foot, what is the height *h* above water that a skier leaves the ramp?

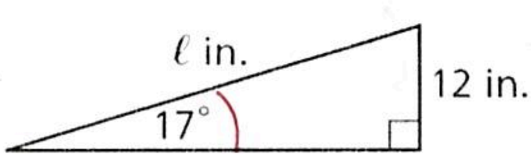


$(19)\sin(15.1) = \frac{h}{\cancel{19}}(19)$

$(19)\sin(15.1) = h$

$\boxed{h = 4.95 \text{ ft}}$

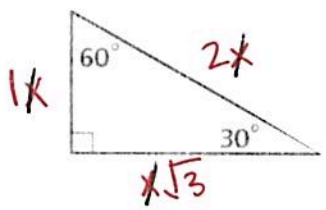
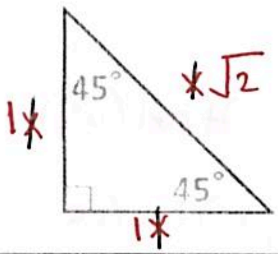
Example 2: Find the length of the skateboard ramp that is lifted 12 in. off the ground at a 17° angle.



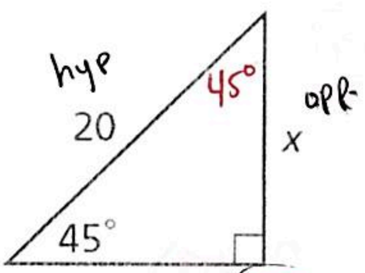
$(h)\sin(17) = \frac{12}{\cancel{h}}(h)$

$\frac{(h)\cancel{\sin(17)}}{\cancel{\sin(17)}} = \frac{12}{\sin(17)}$

$h = \frac{12}{\sin(17)}$ $\boxed{h = 41.04 \text{ in.}}$

Trigonometric Ratios of Special Right Triangles			
Diagram	**Sine**	**Cosine**	**Tangent**
	$\sin 30^\circ = \frac{1}{2}$ $\frac{O}{H}$ $\sin 60^\circ = \frac{\sqrt{3}}{2}$ $\frac{O}{H}$	$\cos 30^\circ = \frac{\sqrt{3}}{2}$ $\frac{A}{H}$ $\cos 60^\circ = \frac{1}{2}$ $\frac{A}{H}$	$\tan 30^\circ = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ $\frac{O}{A}$ $\tan 60^\circ = \frac{\sqrt{3}}{1} = \sqrt{3}$ $\frac{O}{A}$
	$\sin 45^\circ = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\frac{O}{H}$	$\cos 45^\circ = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\frac{A}{H}$	$\tan 45^\circ = \frac{1}{1} = 1$

Find all missing side lengths using each of the trig ratios:



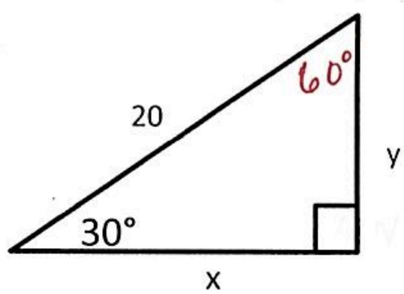
~~$\tan(45) = \frac{1}{1}$~~ $= \frac{x}{x}$

$\sin(45) = \frac{\sqrt{2}}{2} = \frac{x}{20}$

$\cancel{2}x = \frac{20\sqrt{2}}{2}$

$\boxed{x = 10\sqrt{2}}$

$\cos(45) =$ same



$\sin(30) = \frac{1}{2} = \frac{y}{20}$

$2y = 20$

$\boxed{y = 10}$

$\cos(60) = \frac{1}{2} = \frac{y}{20}$

same

$\cos(30) = \frac{\sqrt{3}}{2} = \frac{x}{20}$

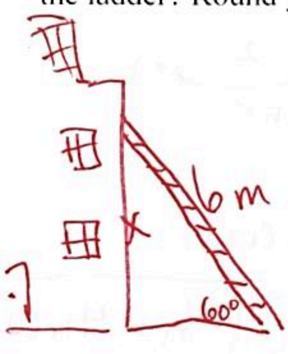
$\frac{20\sqrt{3}}{2} = \frac{\cancel{2}x}{\cancel{2}}$

$\boxed{x = 10\sqrt{3}}$

same

$\sin(60) = \frac{\sqrt{3}}{2} = \frac{x}{20}$

Example 3: A six-meter-long ladder leans against a building. If the ladder makes an angle of 60° with the ground, how far up the wall does the ladder reach? How far from the wall is the base of the ladder? Round your answers to two decimal places, as needed.



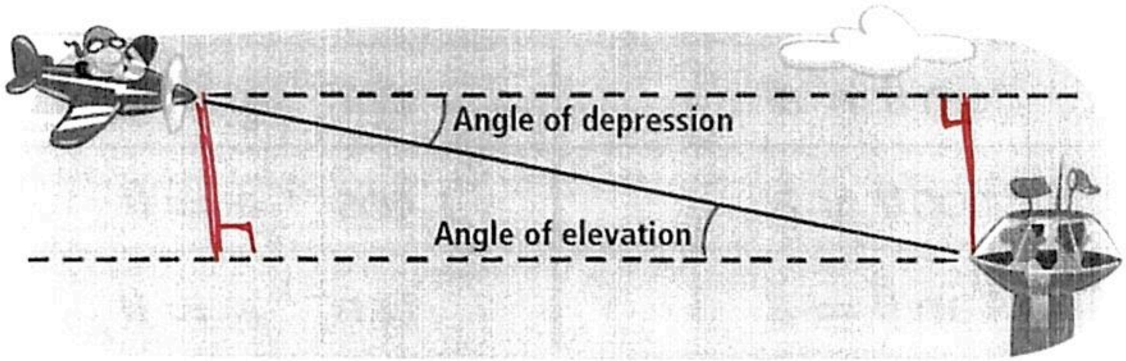
$\sin(60) = \frac{\sqrt{3}}{2} = \frac{x}{6}$

$\cancel{2}x = \frac{6\sqrt{3}}{2}$

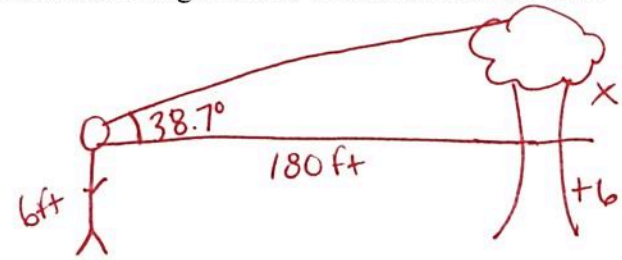
$\boxed{x = 3\sqrt{3}}$

Angle of elevation and depression:

When an object is above or below another object, you can find distances indirectly by using the **angle of elevation** or the **angle of depression** between the objects. Trigonometric ratios are used to solve for these measurements.



Example 4: A biologist whose eye level is 6 *ft* above the ground measures the angle of elevation to the top of a tree to be 38.7 degrees. If the biologist is standing 180 *ft* from the tree's base, what is the height of the tree to the nearest foot?



$$(180)\tan(38.7) = \frac{x}{180}(180)$$

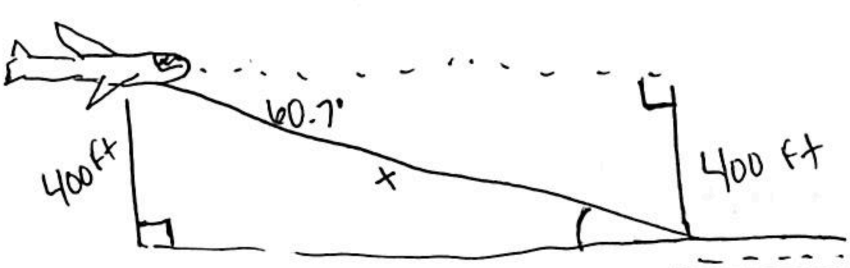
$$x = 180(\tan(38.7))$$

$$x = 144.21$$

$$+6$$

$$\boxed{150.21 \text{ ft}}$$

Example 5: A ~~surveyor whose eye level is 6 *ft*~~ pilot who is 400ft above the ground measures the angle of ~~elevation~~ depression to ~~the top of the highest hill on a roller coaster~~ to be 60.7 degrees. ~~If the surveyor is standing 120 *ft* from the hill's base, what is the height of the hill to the nearest foot?~~ How far is he from the landing strip.



$$\sin(60.7) = \frac{400}{x}$$

$$\frac{400}{\sin(60.7)} = x$$

$$\boxed{x = 458.68 \text{ ft}}$$

Inverse Trig:

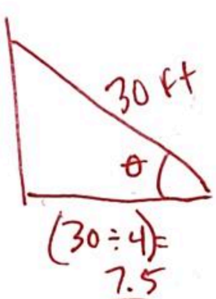
You have evaluated trigonometric functions for a given angle. You can also find the measure of angles given the value of a trigonometric function by using an **inverse trigonometric** relation.

Function	Inverse Relation
$\sin \theta = a$	$\sin^{-1} a = \theta$
$\cos \theta = a$	$\cos^{-1} a = \theta$
$\tan \theta = a$	$\tan^{-1} a = \theta$

Reading Math

The expression $\sin^{-1}$ is read as "the inverse sine." In this notation, $^{-1}$ indicates the *inverse* of the sine function, NOT the *reciprocal* of the sine function.

Example 6: A painter needs to lean a 30 *ft* ladder against a wall. Safety guidelines recommend that the distance between the base of the ladder and the wall should be ¼ of the length of the ladder. To the nearest degree, what acute angle should the ladder make with the ground?

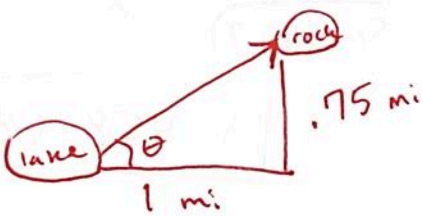


$\cos \theta = \frac{7.5}{30}$

$\cos^{-1}\left(\frac{7.5}{30}\right) = \theta$

$\theta = 75.52 \approx \boxed{76°}$

Example 7: A group of hikers wants to walk from a lake to an unusual rock formation. The formation is 1 mile east and 0.75 mile north of the lake. To the nearest degree, in what direction should the hikers head from the lake to reach the rock formation?

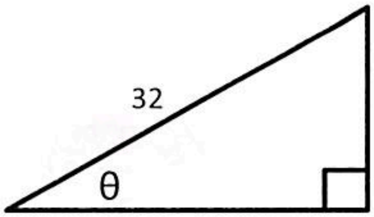


$\tan \theta = \frac{.75}{1}$

$\tan^{-1}(.75) = \theta$

$\theta = 36.87 \approx \boxed{37°}$

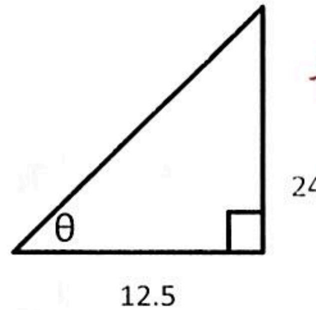
Example 8: Find the value of θ in each triangle below.



$\sin\theta = \frac{19}{32}$

$\sin^{-1}\left(\frac{19}{32}\right) = \theta$

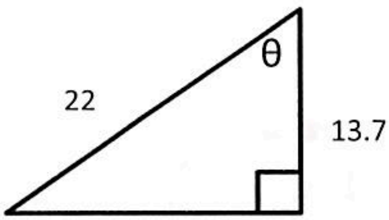
$\theta =$



$\tan\theta = \frac{24}{12.5}$

$\tan^{-1}\left(\frac{24}{12.5}\right) = \theta$

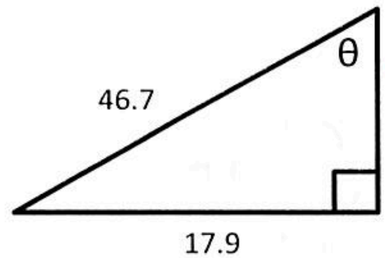
$\theta =$



$\cos\theta = \frac{13.7}{22}$

$\cos^{-1}\left(\frac{13.7}{22}\right) = \theta$

$\theta =$



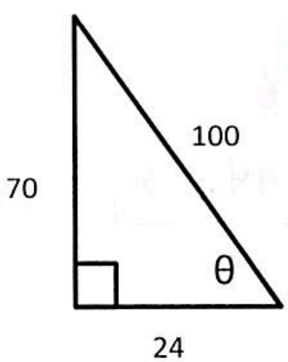
$\sin\theta = \frac{17.9}{46.7}$

$\sin^{-1}\left(\frac{17.9}{46.7}\right) = \theta$

$\theta =$

Extra Problems:

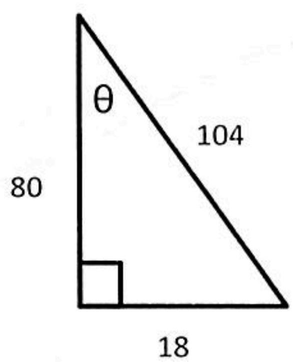
Example 9: Find the ~~values~~ ratios of the three trigonometric functions for θ.



$\sin\theta = \frac{70}{100} = \frac{7}{10}$

$\cos\theta = \frac{24}{100} = \frac{6}{25}$

$\tan\theta = \frac{70}{24} = \frac{35}{12}$



$\sin\theta = \frac{18}{104} = \frac{9}{52}$

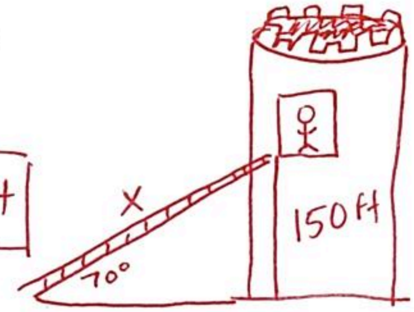
$\cos\theta = \frac{80}{104} = \frac{10}{13}$

$\tan\theta = \frac{18}{80} = \frac{9}{40}$

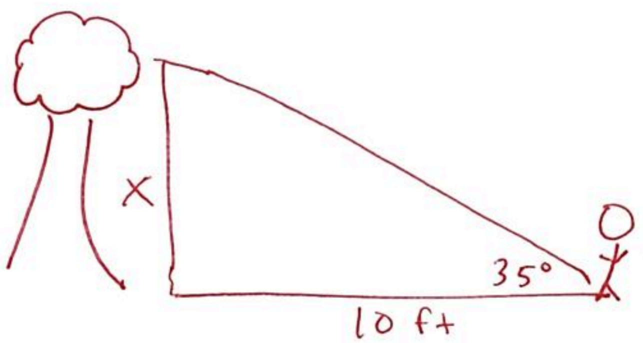
Example 10: Prince Charming is planning to rescue Rapunzel who recently received a haircut. If it is 150 ft from the ground to the bottom of her window and Charming plans to lean his ladder at a 70° angle so he won't fall over, how long of a ladder should he bring?

$$\sin(70) = \frac{150}{x}$$

$$x = \frac{150}{\sin(70)} \qquad \boxed{x = 159.63\ ft}$$



Example 11: Mr. Fitz was trying to find the height of a tree. He determined that the angle of elevation to the top of the tree was 35° at a distance of 10 feet from the base of the tree. What is the height of the tree?

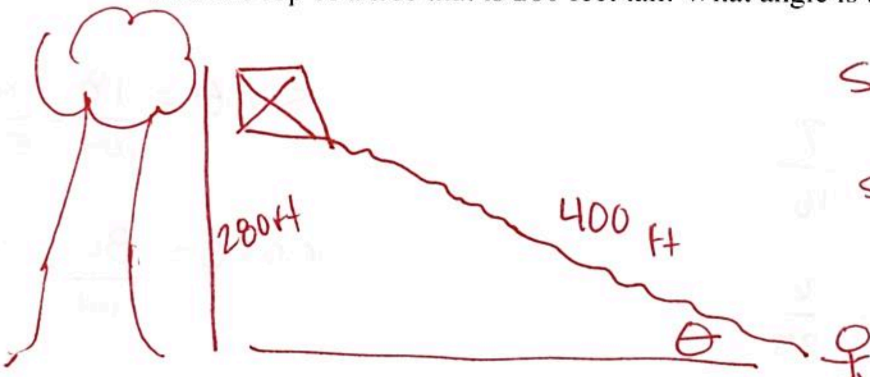


$$\tan(35) = \frac{x}{10}$$

$$(10)\tan(35) = x$$

$$\boxed{x = 7\ ft}$$

Example 12: Megan is flying a kite that has a 400 ft long string. If she sees that the kite is level with the top of a tree that is 280 feet tall. What angle is the kite string forming with the ground?

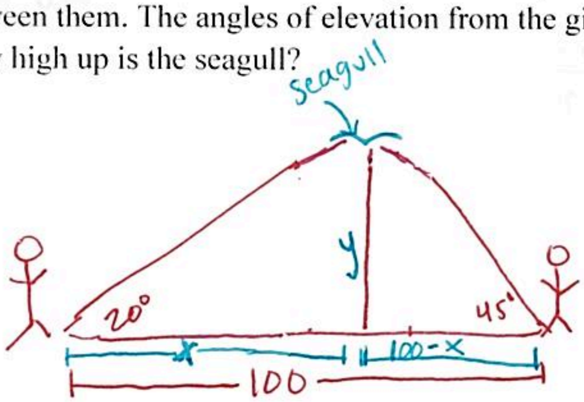


$$\sin\theta = \frac{280}{400}$$

$$\sin^{-1}\left(\frac{280}{400}\right) = \theta$$

$$\boxed{\theta = 44.43°}$$

Example 13: Two girls are standing 100 feet apart. They both see a beautiful seagull in the air between them. The angles of elevation from the girls to the bird are 20° and 45°, respectively. How high up is the seagull?



$$\tan(20) = \frac{y}{x} \qquad \tan(45) = \frac{y}{100-x}$$

$$y = x \cdot \tan(20) \qquad \tan(45) = \frac{.364x}{100-x}$$

$$y = (x \cdot (.364)) \qquad 1 = \frac{.364x}{100-x}$$

$$.364x = 100 - x$$

$$1.364x = 100$$

$$x = 73.31$$

$$y = (73.31)(.364)$$

$$\boxed{y = 26.68\ ft}$$