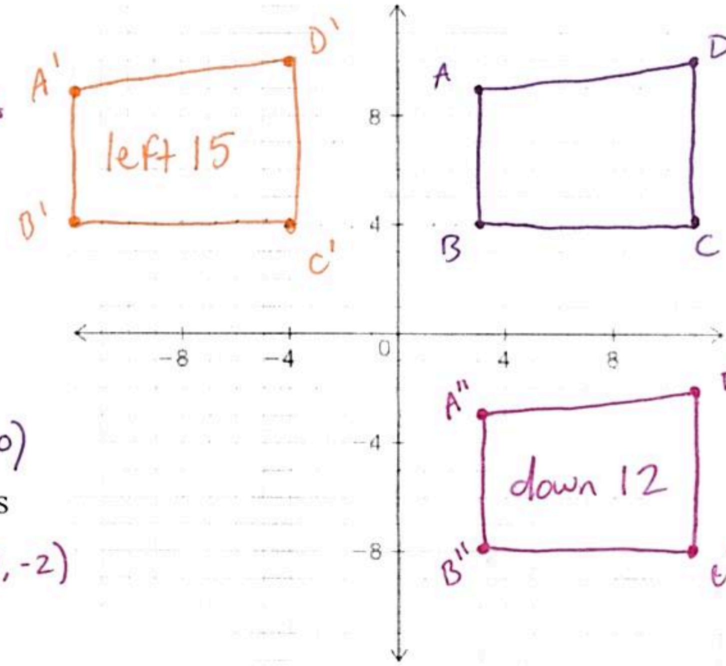
Coach C's Notes

7.1 Translating, Rotating, and Reflecting Geometric Figures



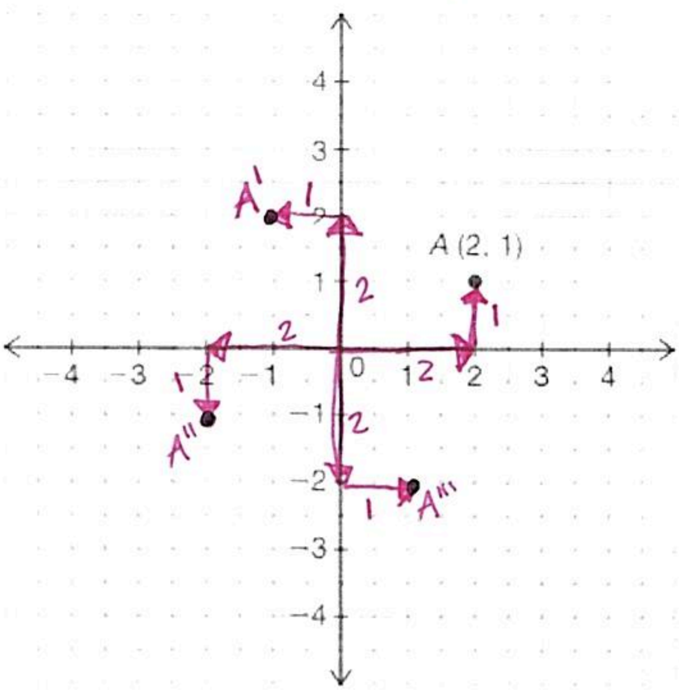
Translating:

Transformation where all points "slide" in one direction

Graph trapezoid ABCD with points
A (3, 9) B (3,4) C (11, 4) D (11, 10)

a. Horizontally translate figure ABCD left 15 units
A' (-12, 9) B' (-12, 4) C' (-4, 4) D' (-4, 10)

b. Vertically translate the image down 12 units
A'' (3, -3) B'' (3, -8) C'' (11, -8) D'' (11, -2)



Rotating:

Transformation that turns a figure about a fixed point called "point of rotation"

Point A is located at (2, 1).

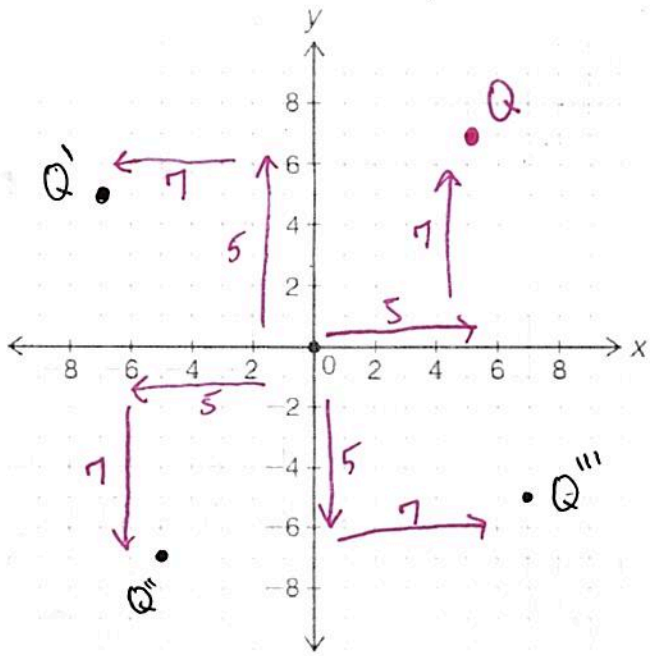
a. Rotate point A 90° counterclockwise about the origin and label it A' (-1, 2)

b. Rotate point A' 90° counterclockwise about the origin and label it A'' (-2, -1)

c. Rotate point A'' 90° counterclockwise about the origin and label it A''' (1, -2)

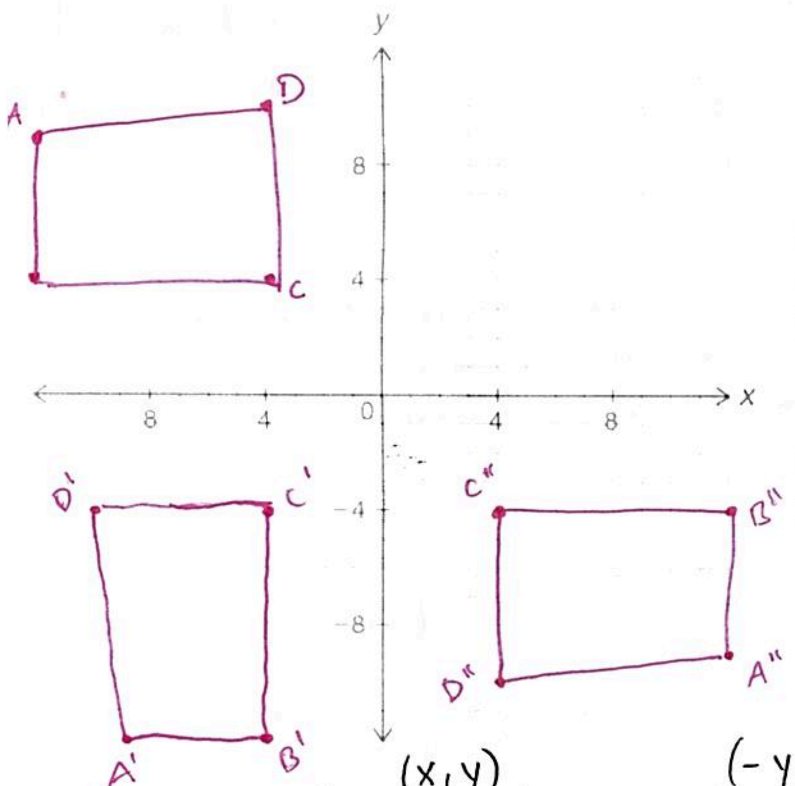
Original Point	Coordinates After a 90° Counterclockwise Rotation About the Origin	Coordinates After a 180° Counterclockwise Rotation About the Origin	Coordinates After a 270° Counterclockwise Rotation About the Origin	Coordinates After a 360° Counterclockwise Rotation About the Origin
(2, 1)	(-1, 2)	(-2, -1)	(1, -2)	(2, 1)
(x, y)	$(-y, x)$	$(-x, -y)$	$(y, -x)$	(x, y)

Graph and label point Q at (5,7). Using the origin as the center of rotation, rotate point Q 90°, 180°, and 270° counterclockwise, labeling accordingly.



Graph trapezoid ABCD with points
A (-12, 9) B (-12, 4)
C (-4, 4) D (-4, 10)

Rotate the shape 90° and 180° counterclockwise about the origin



(x, y) Coordinates of Trapezoid *ABCD*	(-y, x) Coordinates of Trapezoid *A'B'C'D'*	(-x, -y) Coordinates of Trapezoid *A"B"C"D"*
A (–12, 9)	(-9, -12)	(12, -9)
B (–12, 4)	(-4, -12)	(12, -4)
C (–4, 4)	(-4, -4)	(4, -4)
D (–4, 10)	(-10, -4)	(4, -10)

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Rotations without graphing… * (x, y)

The vertices of parallelogram DEFG are D (-9, 7), E (-12, 2), F (-3, 2), and G (0, 7).

a. Determine the vertex coordinates of image D'E'F'G' if parallelogram DEFG is rotated 90° counterclockwise about the origin.

*(-y, x) D' (-7, -9) E' (-2, -12) F' (-2, -3) G' (-7, 0)

b. Determine the vertex coordinates of image D''E''F''G'' if parallelogram DEFG is rotated 180° counterclockwise about the origin.

*(-x, -y) D'' (9, -7) E'' (12, -2) F'' (3, -2) G'' (0, -7)

c. Determine the vertex coordinates of image D'''E'''F'''G''' if parallelogram DEFG is rotated 270° counterclockwise about the origin.

*(y, -x) D''' (7, 9) E''' (2, 12) F''' (2, 3) G''' (7, 0)

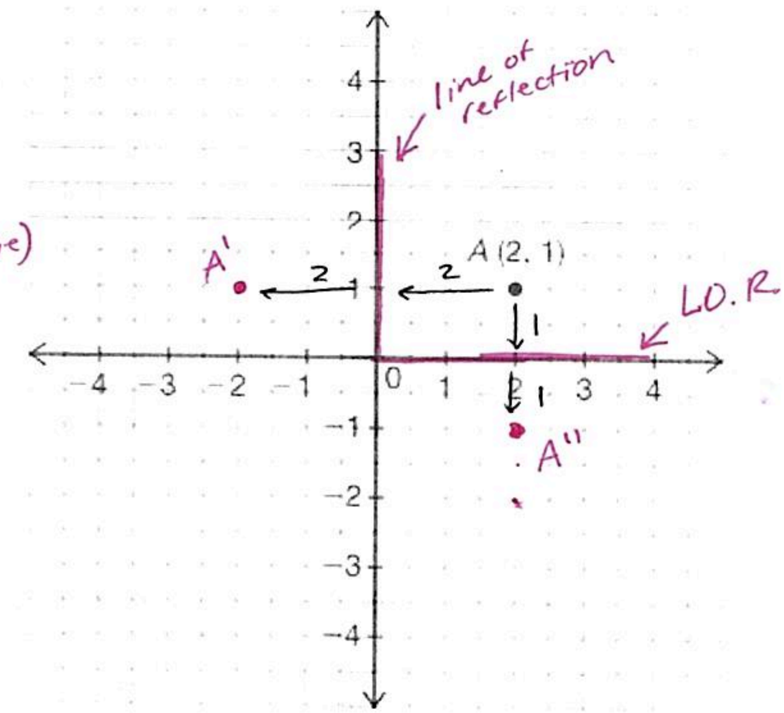
Reflecting:

Transformation that "flips" a figure over a given line called "line of reflection" (mirror image)

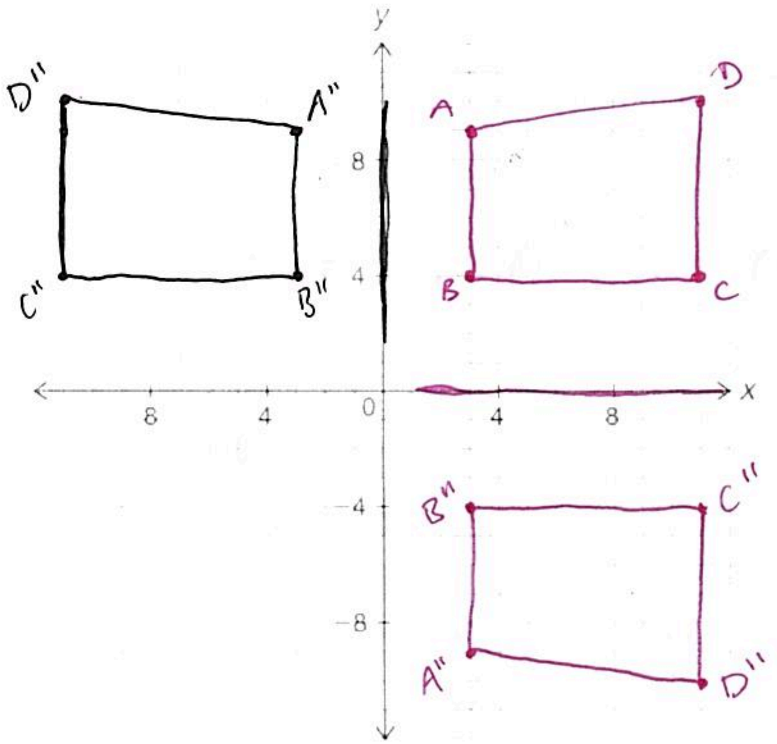
Point A is located at (2, 1).

a. Reflect point A over the y-axis and label it A'

b. Reflect point A over the x-axis and label it A''



Original Point	Coordinates of Image After a Reflection Over the *x*-axis	Coordinates of Image After a Reflection Over the *y*-axis
(2, 1)	(2, -1)	(-2, 1)
(*x*, *y*)	(x, -y)	(-x, y)



Graph trapezoid ABCD with points
A (3, 9) B (3, 4)
C (11, 4) D (11, 10)

Reflect the shape over the x-axis to form trapezoid A'B'C'D'

Reflect the shape over the ~~x~~ Y-axis to form trapezoid A"B"C"D"

(x, y) Coordinates of Trapezoid *ABCD*	(x, -y) Coordinates of Trapezoid *A'B'C'D'*	(-x, y) Coordinates of Trapezoid *A"B"C"D"*
A (3, 9)	(3, -9)	(-3, 9)
B (3, 4)	(3, -4)	(-3, 4)
C (11, 4)	(11, -4)	(-11, 4)
D (11, 10)	(11, -10)	(-11, 10)

Reflections without graphing… * (x, y)

The vertices of parallelogram DEFG are D (-9, 7), E (-12, 2), F (-3, 2), and G (0, 7).

a. Determine the vertex coordinates of image D'E'F'G' if parallelogram DEFG is reflected over the x-axis.

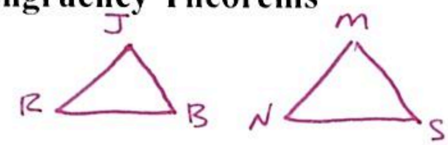
* (x, -y) D' (-9, -7) E' (-12, -2) F' (-3, -2) G' (0, -7)

b. Determine the vertex coordinates of image D''E''F''G'' if parallelogram DEFG is reflected over the y-axis.

* (-x, y) D" (9, 7) E" (12, 2) F" (3, 2) G" (0, 7)

7.2 - 7.6 Congruent Triangles and Congruency Theorems

Consider the <u>congruence statement</u> $\Delta JRB \cong \Delta MNS$

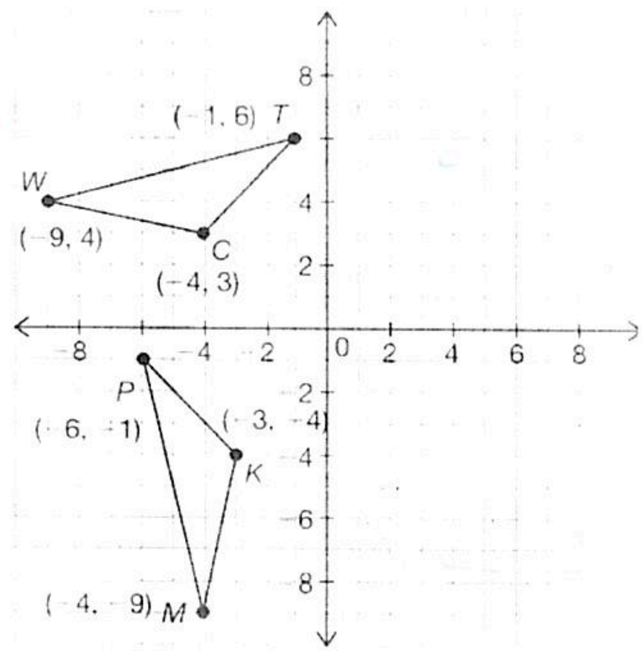


a. Identify the congruent angles

$\angle J \cong \angle M$
$\angle R \cong \angle N$
$\angle B \cong \angle S$

b. Identify the congruent sides

$\overline{JR} \cong \overline{MN}$ $\overline{JB} \cong \overline{MS}$
$\overline{RB} \cong \overline{NS}$



a. Determine the transformation used to create ΔPMK.

ΔTWC was rotated 90° counterclockwise

b. Does the transformation preserve the size and shape of the triangle in this problem situation? Why or why not?

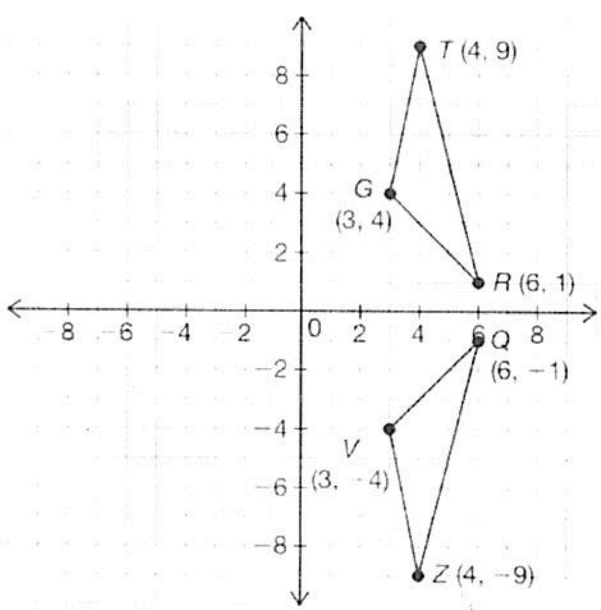
Yes, rotations don't affect size/shape

c. Write a triangle congruence statement for the triangles.

$\Delta TWC \cong \Delta PMK$

d. Identify the congruent angles and congruent sides.

$\angle T \cong \angle P$ $\overline{TW} \cong \overline{PM}$
$\angle W \cong \angle M$ $\overline{WC} \cong \overline{MK}$
$\angle C \cong \angle K$ $\overline{TC} \cong \overline{PK}$



a. Determine the transformation used to create ΔZQV.

ΔTRG was reflected over x-axis

b. Does the transformation preserve the size and shape of the triangle in this problem situation? Why or why not?

Yes, reflections don't affect size/shape

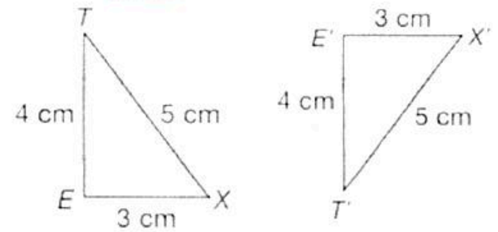
c. Write a triangle congruence statement for the triangles.

$\Delta TRG \cong \Delta ZQV$

d. Identify the congruent angles and congruent sides.

$\angle T \cong \angle Z$ $\overline{TR} \cong \overline{ZQ}$
$\angle R \cong \angle Q$ $\overline{RG} \cong \overline{QV}$
$\angle G \cong \angle V$ $\overline{TG} \cong \overline{ZQ}$

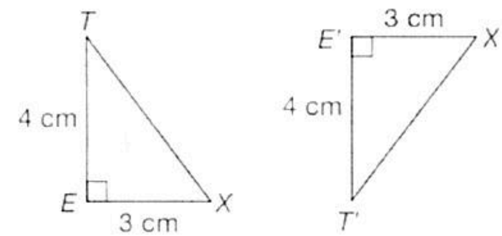
Side-Side-Side Congruence theorem: "If three sides of one triangle are congruent to the corresponding sides of another triangle, then the triangles are congruent."



** To prove that two triangles are congruent using the Side-Side-Side Congruence Theorem, you need to know the lengths of the corresponding sides of the triangles

$\Delta TEX \cong \Delta T'E'X'$ by the Side-Side-Side Congruence Theorem

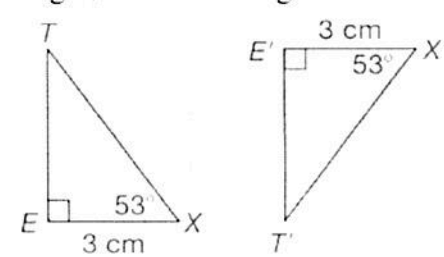
Side-Angle-Side Congruence theorem: "If two sides and the included angle of one triangle are congruent to the corresponding sides and the included angle of the second triangle, then the triangles are congruent."



** To prove that two triangles are congruent using the Side-Angle-Side Congruence Theorem, you need to know the lengths of two pairs of corresponding sides and the measure of the corresponding included angles

$\Delta TEX \cong \Delta T'E'X'$ by the Side-Angle-Side Congruence Theorem

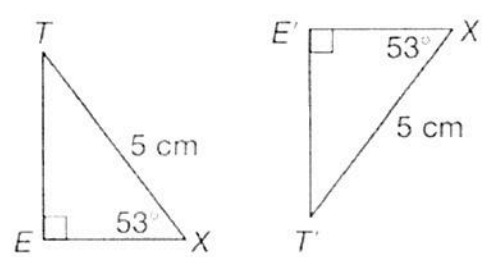
Angle-Side-Angle Congruence theorem: "If two angles and the included side of one triangle are congruent to the corresponding two angles and the included side of another triangle, then the triangles are congruent."



**To prove that two triangles are congruent using the Angle-Side-Angle Congruence Theorem, you need to know the measures of two pairs of corresponding angles of the triangles and the measure of the corresponding included side length

$\Delta TEX \cong \Delta T'E'X'$ by the Angle-Side-Angle Congruence Theorem

Angle-Angle-Side Congruence theorem: "If two angles and a non-included side of one triangle are congruent to the corresponding angles and the corresponding non-included side of a second triangle, then the triangles are congruent."

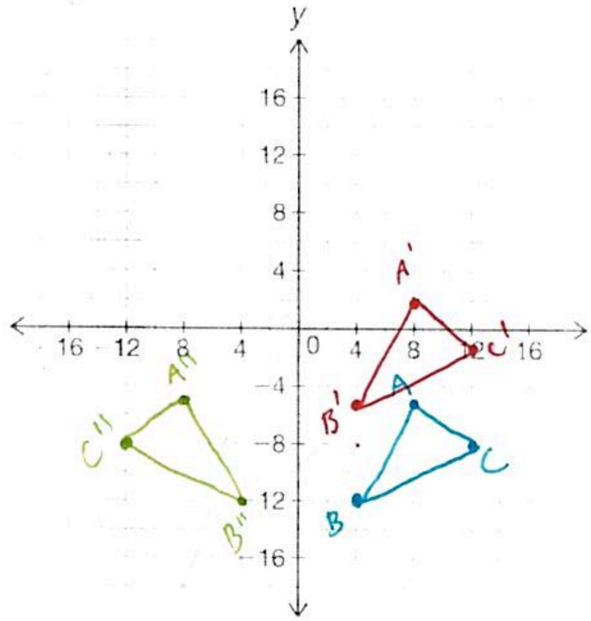


**To prove that two triangles are congruent using the Angle-Angle-Side Congruence Theorem, you need to know the measures of two corresponding angles of the triangles and the measure of a pair of corresponding non-included side lengths.

$\Delta TEX \cong \Delta T'E'X'$ by the Angle-Angle-Side Congruence Theorem

SSS

1. Graph triangle ABC by plotting the points A (8, -5), B (4, -12), and C (12, -8).
   a. Calculate the length of each side of the triangle.
2. Translate line segments $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ up 7 units to form triangle A'B'C'.
   a. Calculate the length of each side of the triangle.
3. Reflect line segments $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ over the y-axis form triangle A''B''C''.
   a. Calculate the length of each side of the triangle.



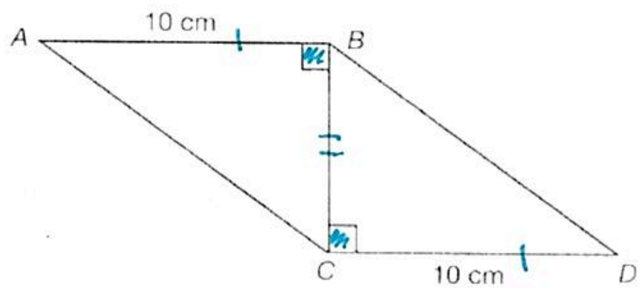
4. Are the triangles congruent? Explain.

Yes. The side lengths are congruent after each transformation so the Δ's are ≅ with SSS.

Δ ABC	Length of side	Δ A'B'C'	Length of side	Δ A''B''C''	Length of side
$\overline{AB}$	$\sqrt{65}$	$\overline{A'B'}$	$\sqrt{65}$	$\overline{A''B''}$	$\sqrt{65}$
$\overline{BC}$	$\sqrt{80}$	$\overline{B'C'}$	$\sqrt{80}$	$\overline{B''C''}$	$\sqrt{80}$
$\overline{AC}$	5	$\overline{A'C'}$	5	$\overline{A''C''}$	5

SAS

Write the three congruence statements that show ΔABC ≅ ΔDCB by the SAS Congruence Theorem.

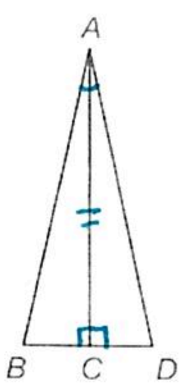


$\overline{AB} \cong \overline{DC}$

$\angle ABC \cong \angle DCB$

$\overline{BC} \cong \overline{BC}$

ASA



Suppose ~~$\overline{AD}$~~ $\overline{AC}$ ⊥ $\overline{BC}$, and ~~$\overline{AD}$~~ $\overline{AC}$ bisects ∠A

Are there congruent triangles in the diagram? Explain.

Yes.

$\angle DAC \cong \angle BAC$

$\overline{AC} \cong \overline{AC}$

$\angle DCA \cong \angle BCA$

AAS

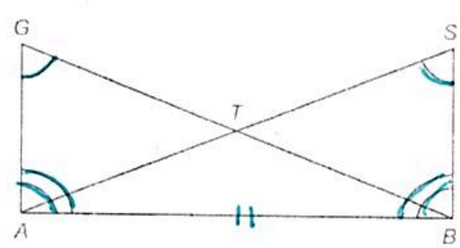
Is ΔGAB ≅ ΔSBA? Explain.

Yes

$\angle BGA \cong \angle ASB$

$\angle GAB \cong \angle SBA$

$\overline{AB} \cong \overline{AB}$



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7.7 Using Congruent Triangles (Practice Problems)

Determine if the triangles are congruent and by which theorem? Justify your reasoning.

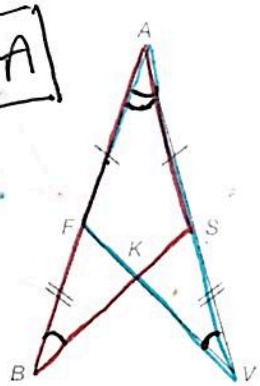
$\triangle ABS \stackrel{?}{\cong} \triangle AVF$

ASA

$\overline{AB} \cong \overline{VA}$

$\angle B \cong \angle V$

$\angle A \cong \angle A$



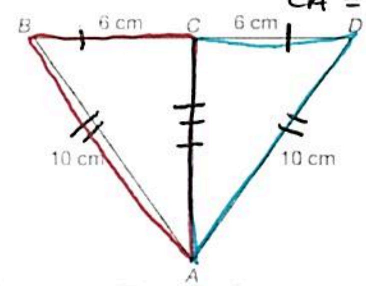
$\triangle ABC \stackrel{?}{\cong} \triangle ADC$

SSS

$\overline{BC} \cong \overline{DC}$

$\overline{BA} \cong \overline{DA}$

$\overline{CA} \cong \overline{CA}$



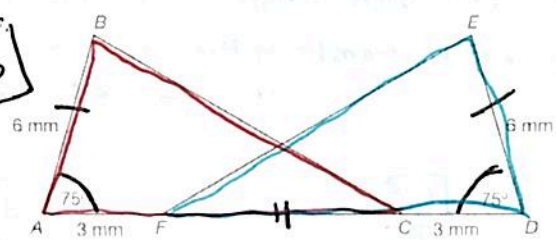
$\triangle ABC \stackrel{?}{\cong} \triangle DEF$

SAS

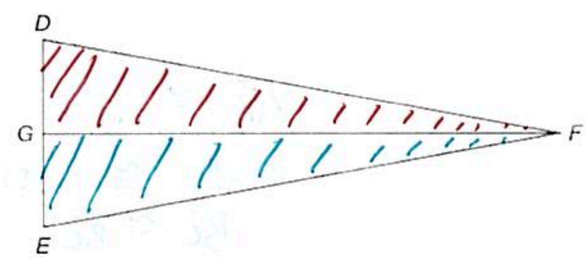
$\overline{AC} \cong \overline{DF}$

$\angle A \cong \angle D$

$\overline{AB} = \overline{DE}$



Determine which given information results in $\triangle DFG \cong \triangle EFG$. State the appropriate congruence theorem if the triangles can be proven congruent, or state that there is not enough information if additional givens are needed to determine congruent triangles.



1. Given: G is the midpoint of $\overline{DE}$

   Not enough information to determine $\cong \triangle$'s

2. Given: $\overline{DE} \perp \overline{FG}$

   Not enough information to determine $\cong \triangle$'s

3. Given: $\overline{FG}$ bisects $\angle DFE$, $\overline{FD} \cong \overline{FE}$

   $\triangle DFG \cong \triangle EFG$ using SAS

4. Given: ΔDEF is isosceles with $\overline{FD} \cong \overline{FE}$

   Not enough information to determine $\cong \triangle$'s

5. Given: $\overline{FG}$ bisects $\angle DFE$, $\angle DGF$ is a right angle

   $\triangle DFG \cong \triangle EFG$ using ASA

6. Given: $\angle D \cong \angle E$, $\overline{FG}$ bisects $\angle DFE$

   $\triangle DFG \cong \triangle EFG$ using AAS

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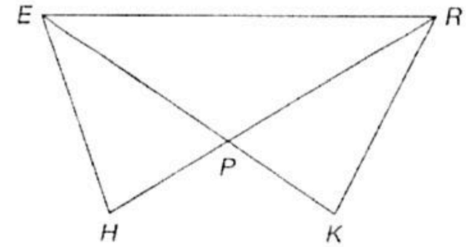
1. Determine what additional information is needed to prove the specified triangles congruent.



a. Given: $\angle H \cong \angle K$
Prove: $\Delta EPH \cong \Delta RPK$ by ASA Congruence Theorem

$\overline{HP} \cong \overline{KP}$

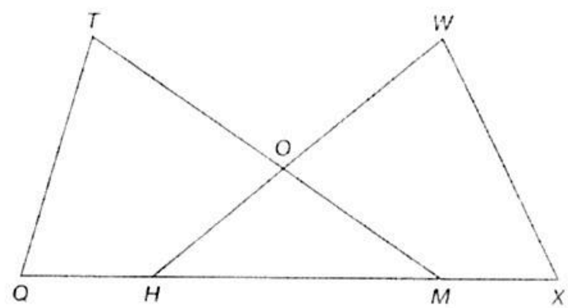
b. Given: $\overline{HE} \cong \overline{KR}$
Prove: $\Delta EHR \cong \Delta RKE$ by SSS Congruence Theorem

$\overline{HR} \cong \overline{KE}$

c. Given: ΔEPR is isosceles with $\overline{EP} \cong \overline{RP}$
Prove: $\Delta EPH \cong \Delta RPK$ by SAS Congruence Theorem

$\overline{HP} \cong \overline{KP}$

2. Determine what additional information is needed to prove the specified triangles congruent.



a. Given: $\overline{QH} \cong \overline{XM}$, $\overline{TQ} \cong \overline{WX}$
Prove: $\Delta TQM \cong \Delta WXH$ by SAS Congruence Theorem

$\angle Q \cong \angle X$

b. Given: $\angle Q \cong \angle X$, $\angle T \cong \angle W$
Prove: $\Delta TQM \cong \Delta WXH$ by AAS Congruence Theorem

$\overline{TM} \cong \overline{WH}$ or $\overline{QM} \cong \overline{XH}$