# Looking at Something Familiar in a New Way

3.2

## Area and Perimeter of Triangles on the Coordinate Plane

### LEARNING GOALS

In this lesson, you will:

- Determine the perimeter of triangles on the coordinate plane.
- Determine the area of triangles on the coordinate plane.
- Determine and describe how proportional and non-proportional changes in the linear dimensions of a triangle affect its perimeter and area.
- Explore the effects that doubling the area has on the properties of a triangle.

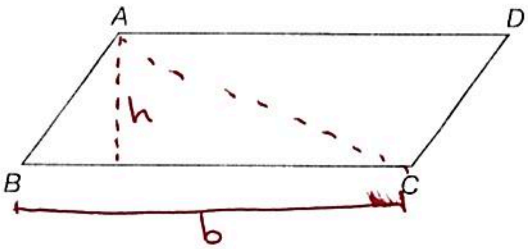
One of the most famous stretches of ocean in the Atlantic is an area that stretches between the United States, Puerto Rico, and Bermuda known as the Bermuda Triangle.

A heavily traveled area by planes and ships, it has become famous because of the many stories about ships and planes lost or destroyed as they moved through the Triangle.

For years, the Bermuda Triangle was suspected of having mysterious, supernatural powers that fatally affected all who traveled through it. Others believe natural phenomena, such as human error and dangerous weather, are to blame for the incidents.

## PROBLEM 1 Determining the Area of a Triangle

1. The formula for calculating the area of a triangle can be determined from the formula for the area of a parallelogram.



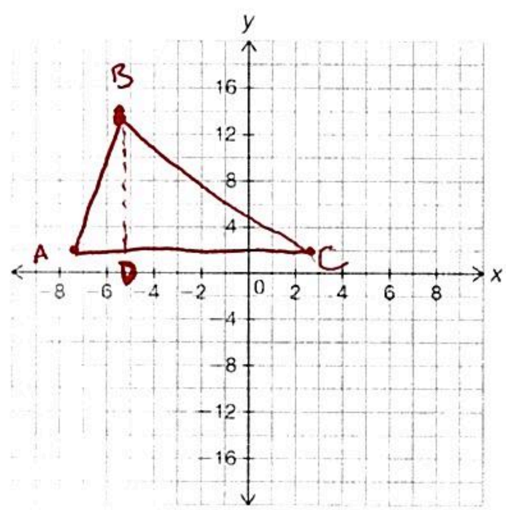

   a. Explain how the formula for the area of a triangle is derived using the given parallelogram.



   b. Write the formula for the area of a triangle.

$$A = \frac{1}{2}bh$$

2. Graph triangle *ABC* with vertices *A*(−7.5, 2), *B*(−5.5, 13), and *C*(2.5, 2). Then, determine its perimeter.




$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{(-5.5-(-7.5))^2 + (13-2)^2}$$
$$= \sqrt{2^2 + 11^2}$$
$$= \sqrt{125} = 5\sqrt{5}$$

$$BC = \sqrt{(2.5-(-5.5))^2 + (2-13)^2}$$
$$= \sqrt{8^2 + -11^2}$$
$$= \sqrt{185}$$

$$AC = 10$$

$$\text{Perimeter} = AB + BC + AC$$

$$P = 5\sqrt{5} + \sqrt{185} + 10$$

$$\boxed{P \approx 34.8}$$

3. Determine the area of triangle *ABC*.

a. What information is needed about triangle *ABC* to determine its area?

length of base

length of height

b. Arlo says that line segment *AB* can be used as the height. Trisha disagrees and says that line segment *BC* can be used as the height. Randy disagrees with both of them and says that none of the line segments that make up the triangle can be used as the height. Who is correct? Explain your reasoning.

c. Draw a line segment that represents the height of triangle *ABC*. Label the line segment *BD*. Then, determine the height of triangle *ABC*.

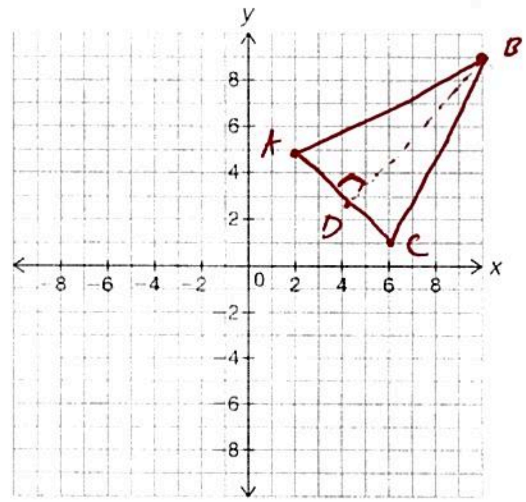
y-coordinate of B is 13, Point D has same y-coordinate as A and C, so it is 2. 13 - 2 = $\boxed{11 \text{ units}}$


d. Determine the area of triangle *ABC*.

$$A = \tfrac{1}{2}bh$$
$$= \tfrac{1}{2}(10)(11)$$
$$\boxed{= 55 \text{ units}^2}$$

## PROBLEM 2 Which Way Is Up?

1. Graph triangle $ABC$ with vertices $A(2, 5)$, $B(10, 9)$, and $C(6, 1)$. Determine the perimeter.




* use distance formula between points

$AB = 4\sqrt{5}$ $= 8.94$

$BC = 4\sqrt{5}$ $= 8.94$

$AC = 4\sqrt{2}$ $= 5.66$ ← *Base*

Perimeter $= AB + BC + AC$

$= 4\sqrt{5} + 4\sqrt{5} + 4\sqrt{2}$

$\boxed{= 23.5}$

2. To determine the area, you will need to determine the height. How will determining the height of this triangle be different from determining the height of the triangle in Problem 1?

The height has a slope and it is not just a vertical line.

To determine the height of this triangle, you must first determine the endpoints of the height. Remember that the height must always be perpendicular to the base.

Remember, the slopes of perpendicular lines are negative reciprocals.

Let's use $AC$ as the base of triangle $ABC$. Determine the coordinates of the endpoints of height $BD$.

Calculate the slope of the base.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 5}{6 - 2} = \frac{-4}{4} = -1$$

Determine the slope of the height.

$$m = 1$$

Determine the equation of the base.

Base $AC$ has a slope of $-1$ and passed through point $A$ (2, 5).

$$(y - y_1) = m(x - x_1)$$
$$(y - 5) = -1(x - 2)$$
$$y = -x + 7$$

Determine the equation of the height.

Height $\overline{BD}$ has a slope of 1 and passed through point $B$ (10, 9).

$$(y - y_1) = m(x - x_1)$$
$$(y - 9) = 1(x - 10)$$
$$y = x - 1$$

Solve the system of equations to determine the coordinate of the point of intersection.

$$x - 1 = -x + 7 \qquad y = x - 1$$
$$2x = 8 \qquad y = 4 - 1$$
$$x = 4 \qquad y = 3$$

The coordinates of point $D$ are (4, 3).

Finding height:

① Find $m\overline{AC}$

② Find slope $AC$

③ Find $\perp$ slope to $AC$

④ $y - y_1 = m(x - x_1)$
use $\perp$ slope for $m$ and point $B$.

⑤ Find intersection point of $\overline{AC}$ and height.

⑥ Distance between "B" and new point "D"

3. Graph the point of intersection on the coordinate plane and label it point *D*. Draw line segment *BD* to represent the height.

4. Determine the area of triangle *ABC*.

   a. Determine the length of height $\overline{BD}$.

$$BD = \sqrt{(10-4)^2 + (9-3)^2}$$
$$= \sqrt{72}$$
$$\approx 8.49$$

   b. Determine the area of triangle *ABC*.

$$A = \tfrac{1}{2}bh$$
$$A = \tfrac{1}{2}(5.66)(8.49)$$
$$\boxed{A = 24.03}$$

5. ~~You know~~ that any side of a triangle can be the base of the triangle. Predict whether ~~using~~ a different side as the base will result in a different area of the triangle. Explain your reasoning.

Let's consider your prediction.

6. ~~Triangle~~ *ABC* is given on the coordinate plane. This time, let's consider side $\overline{AB}$ as the base.