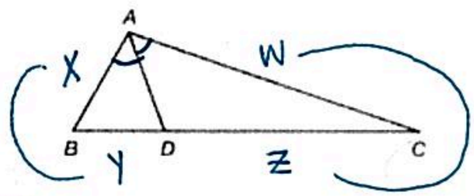
Coach C's Notes

6.3 Theorems about Proportionality

The Angle Bisector/Proportional Side Theorem: "A bisector of an angle in a triangle divides the opposite side into two segments whose lengths are in the same ratio as the lengths of the sides adjacent to the angle."

Given: $\overline{AD}$ bisects $\angle BAC$

Prove: $\frac{AB}{AC} = \frac{BD}{CD}$

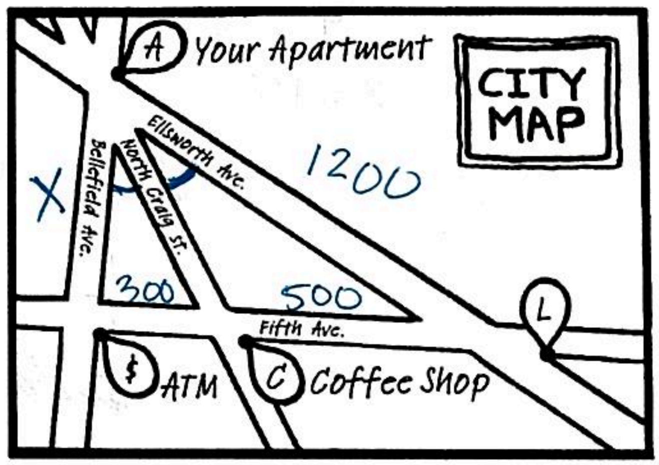


$\frac{x}{w} = \frac{y}{z}$ or $\frac{x}{y} = \frac{w}{z}$

On the map shown, North Craig Street bisects the angle formed between Bellefield Avenue and Ellsworth Avenue.

- The distance from the ATM to the Coffee Shop is 300 feet.
- The distance from the Coffee Shop to the Library is 500 feet.
- The distance from your apartment to the Library is 1200 feet.

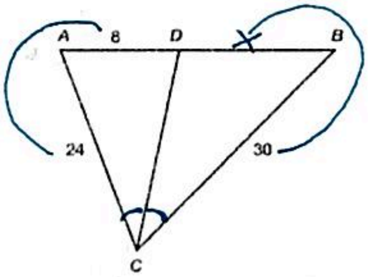
Determine the distance from your apartment to the ATM.



$\frac{x}{300} = \frac{1200}{500}$

$x = 720$ ft

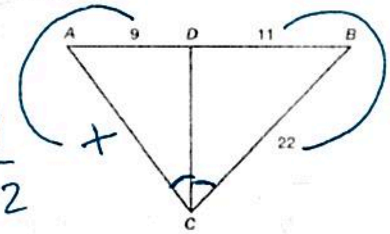
2. *CD* bisects $\angle C$. Solve for *DB*.



$\frac{8}{24} = \frac{x}{30}$

$x = 10$

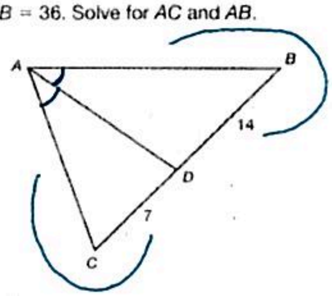
3. *CD* bisects $\angle C$. Solve for *AC*.



$\frac{9}{x} = \frac{11}{22}$

$x = 18$

4. *AD* bisects $\angle A$. $AC + AB = 36$. Solve for *AC* and *AB*.



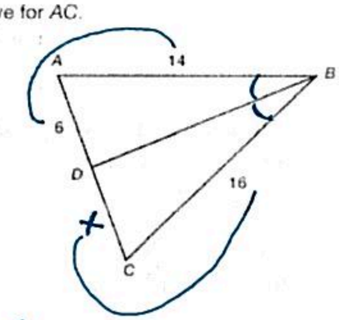
$\frac{AC}{7} = \frac{AB}{14}$

$2AC = AB$

$AC + 2AC = 36$

$AC = 12$ $AB = 24$

5. *BD* bisects $\angle B$. Solve for *AC*.



$\frac{14}{6} = \frac{16}{x}$

$x = 6.9$

$AC = 6 + 6.9$

$AC = 12.6$

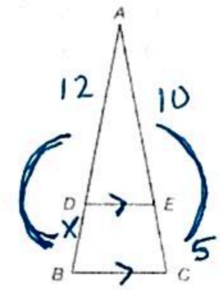
The Triangle Proportionality Theorem: "If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally."

$$\frac{12}{x} = \frac{10}{5}$$

$$x = 6$$

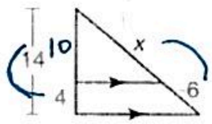
Given: $\overline{BC} \parallel \overline{DE}$

Prove: $\frac{BD}{DA} = \frac{CE}{EA}$



Determine each unknown value.

a.

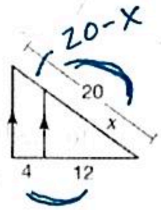


$14 - 4 = 10$

$$\frac{10}{4} = \frac{x}{6}$$

$$x = 15$$

b.



$$\frac{20-x}{x} = \frac{4}{12}$$

$$12(20-x) = 4x$$

$$240 - 12x = 4x$$

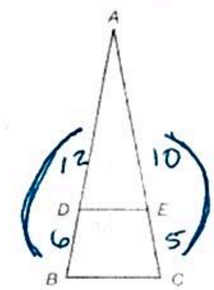
$$240 = 16x$$

$$x = 15$$

The Converse of the Triangle Proportionality Theorem: "If a line divides two sides of a triangle proportionally, then it is parallel to the third side."

Given: $\frac{BD}{DA} = \frac{CE}{EA}$

Prove: $\overline{BC} \parallel \overline{DE}$



$$\frac{12}{6} = \frac{10}{5} \rightarrow \frac{2}{1} = \frac{2}{1} \checkmark$$

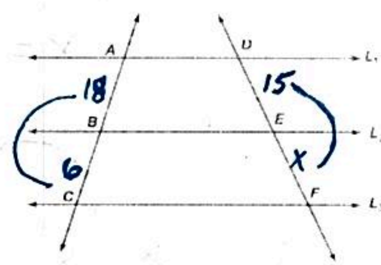
The Proportional Segments Theorem: "If three parallel lines intersect two transversals, then they divide the transversals proportionally."

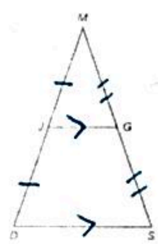
$$\frac{18}{6} = \frac{15}{x}$$

$$x = 5$$

Given: $L_1 \parallel L_2 \parallel L_3$

Prove: $\frac{AB}{BC} = \frac{DE}{EF}$





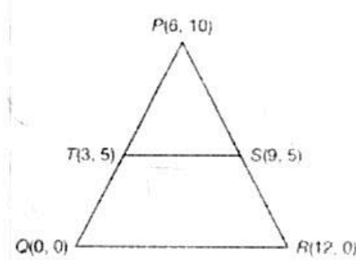
The Triangle Midsegment Theorem: "The midsegment of a triangle is parallel to the third side of the triangle and is half the measure of the third side of the triangle."

JG is midsegment

point J is midpoint of $\overline{MD}$ and

Point G is midpoint of $\overline{MS}$

You can use the Midpoint Formula and the Triangle Midsegment Theorem to verify that two line segments in the coordinate plane are parallel. Consider triangle PQR.



Use the Midpoint Formula to verify that point S is the midpoint of line segment PR and that point T is the midpoint of line segment PQ.

Midpoint formula

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

PQ $\left(\frac{0+6}{2}, \frac{0+10}{2}\right) = (3, 5)$

PR $\left(\frac{12+6}{2}, \frac{0+10}{2}\right) = (9,$