2.2

Special Angles and Postulates

Coach C's Notes

LEARNING GOALS

In this lesson, you will:

- Calculate the complement and supplement of an angle.
- Classify adjacent angles, linear pairs, and vertical angles.
- Differentiate between postulates and theorems.
- Differentiate between Euclidean and non-Euclidean geometries.

KEY TERMS

- supplementary angles
- complementary angles
- adjacent angles
- linear pair
- vertical angles
- postulate
- theorem
- Euclidean geometry
- Linear Pair Postulate
- Segment Addition Postulate
- Angle Addition Postulate

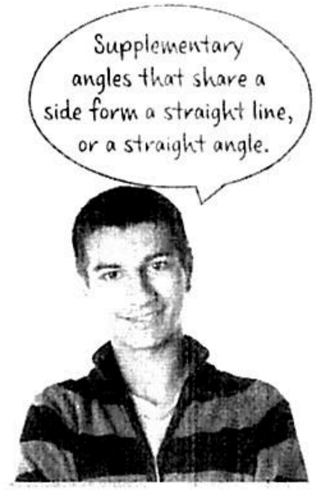
A compliment is an expression of praise, admiration, or congratulations. Often when someone does something noteworthy, you may "pay them a compliment" to recognize the person's accomplishments.

Even though they are spelled similarly, the word "complement" means something very different. To complement something means to complete or to make whole. This phrase is used in mathematics, linguistics, music, and art. For example, complementary angles have measures that sum to 90 degrees—together, they "complete" a right angle. In music, a complement is an interval that when added to another spans an octave—makes it "whole."

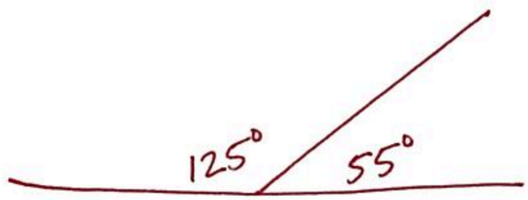
The film *Jerry McGuire* features the famous line "You complete me," meaning that the other person complements them or that together they form a whole. So, a complement can be quite a compliment indeed!

PROBLEM 1 Supplements and Complements

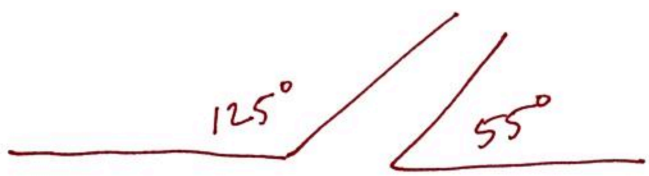
Two angles are **supplementary angles** if the sum of their angle measures is equal to 180°.



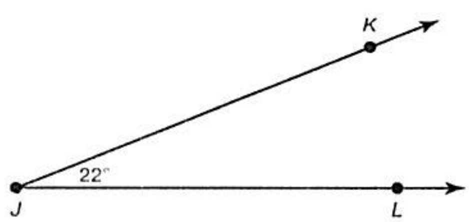
1. Use a protractor to draw a pair of supplementary angles that share a common side, and then measure each angle.



2. Use a protractor to draw a pair of supplementary angles that do not share a common side, and then measure each angle.



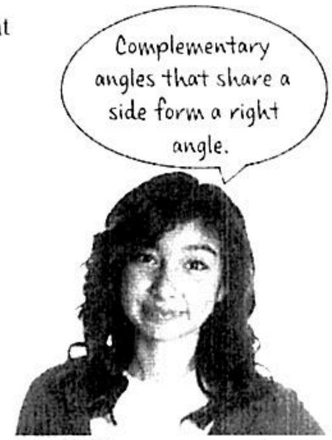
3. Calculate the measure of an angle that is supplementary to $\angle KJL$.

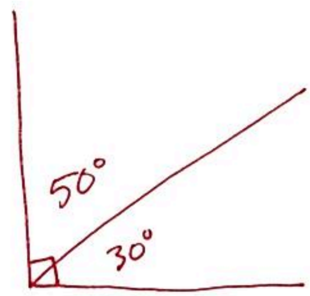


$180 - 22° = \boxed{158°}$

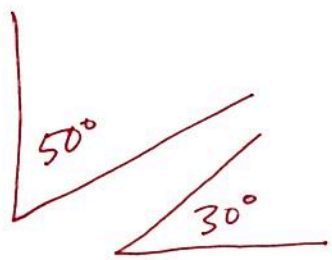
Two angles are **complementary angles** if the sum of their angle measures is equal to 90°.

4. Use a protractor to draw a pair of complementary angles that share a common side, and then measure each angle.

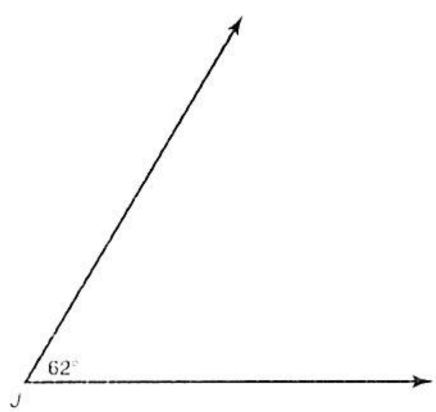




5. Use a protractor to draw a pair of complementary angles that do not share a common side, and then measure each angle.



6. Calculate the measure of an angle that is complementary to ∠J.



$90 - 62 = \boxed{28°}$

7. Determine the measure of each angle. Show your work and explain your reasoning.

a. Two angles are congruent and supplementary.

$$X + X = 180°$$
$$2x = 180°$$
$$X = 90° \,/\, X = 90°$$

b. Two angles are congruent and complementary.

$$X + X = 90°$$
$$2x = 90°$$
$$X = 45° \,/\, x = 45°$$

c. The complement of an angle is twice the measure of the angle.

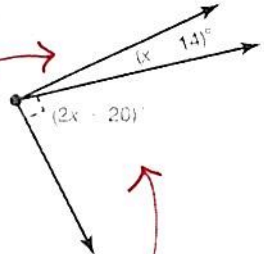
$$X + 2x = 90°$$
$$3x = 90°$$
$$X = 30° \,/\, 60°$$

d. The supplement of an angle is half the measure of the angle.

$$X + 0.5x = 180°$$
$$1.5x = 180°$$
$$X = 120° \,/\, 60°$$

8. Determine the angle measures in each diagram.

a.



$(x-14)+(2x+20)=90^\circ$

$3x+6=90^\circ$

$3x=84^\circ$

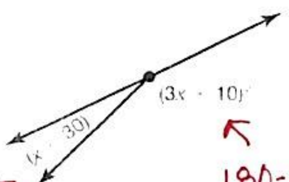
$x=28^\circ$

$(x-14)$

$28-14=\boxed{14^\circ}$

$90^\circ-14^\circ=\boxed{76^\circ}$

b.



$(x-30)+(3x+10)=180^\circ$

$4x-20=180^\circ$

$4x=200^\circ$

$x=50^\circ$

$(x-30)$

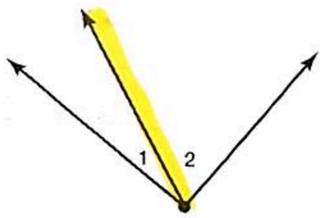
$50-30=\boxed{20^\circ}$

$180-20=\boxed{160^\circ}$

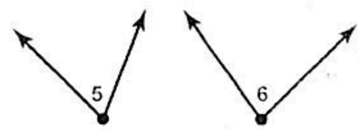
PROBLEM 2 Angle Relationships

You have learned that angles can be supplementary or complementary. Let's explore other angle relationships.

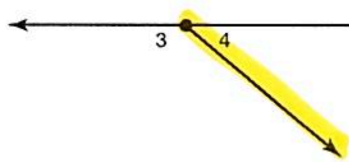
$\angle 1$ and $\angle 2$ are adjacent angles.



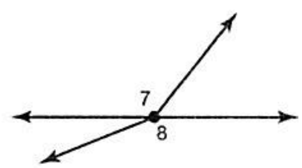
$\angle 5$ and $\angle 6$ are *not* adjacent angles.



$\angle 3$ and $\angle 4$ are adjacent angles.

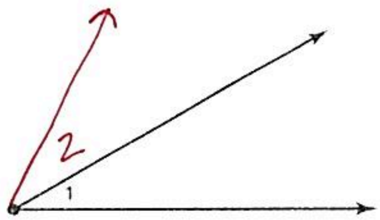


$\angle 7$ and $\angle 8$ are *not* adjacent angles.

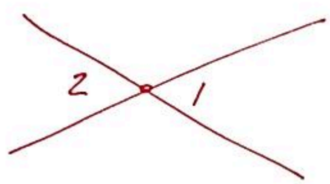


1. Analyze the worked example. Then answer each question.
   a. Describe adjacent angles.

b. Draw $\angle 2$ so that it is adjacent to $\angle 1$.



c. Is it possible to draw two angles that share a common vertex but do not share a common side? If so, draw an example. If not, explain why not.



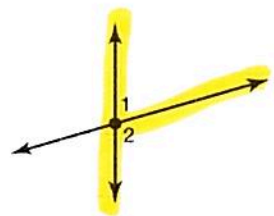
*not adjacent

d. Is it possible to draw two angles that share a common side, but do not share a common vertex? If so, draw an example. If not, explain why not.

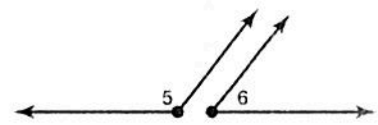
NO

Adjacent angles are two angles that share a common vertex and share a common side.

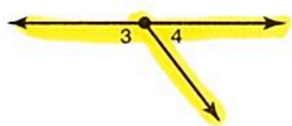
∠1 and ∠2 form a linear pair.



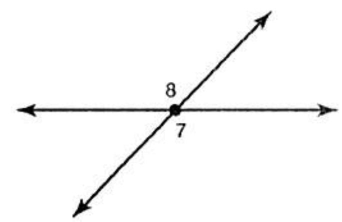
∠5 and ∠6 do *not* form a linear pair.



∠3 and ∠4 form a linear pair.



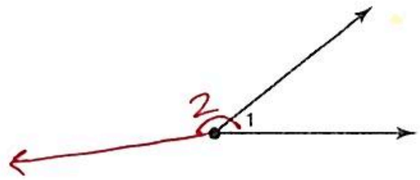
∠7 and ∠8 do *not* form a linear pair.



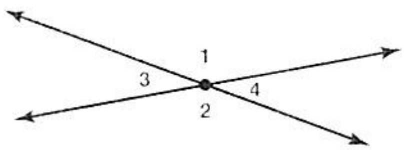
2. Analyze the worked example. Then answer each question.

   a. Describe a linear pair of angles.

b. Draw ∠2 so that it forms a linear pair with ∠1.



c. Name all linear pairs in the figure shown.



1 + 4

4 + 2

2 + 3

3 + 1

d. If the angles that form a linear pair are congruent, what can you conclude?

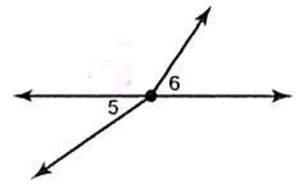
lines are ⊥

angles are ∟

A **linear pair** of angles are two adjacent angles that have noncommon sides that form a line.

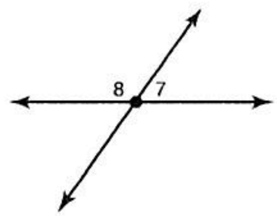
$\angle 1$ and $\angle 2$ are vertical angles.

$\angle 5$ and $\angle 6$ are *not* vertical angles.



$\angle 3$ and $\angle 4$ are vertical angles.

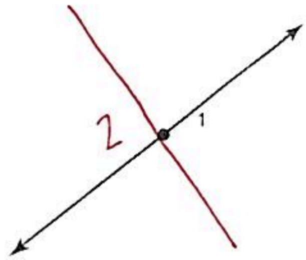
$\angle 7$ and $\angle 8$ are *not* vertical angles.



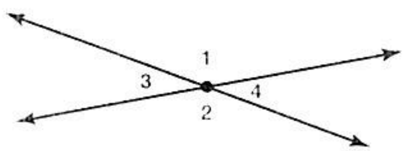
3. Analyze the worked example. Then answer each question.

   a. Describe vertical angles.

b. Draw ∠2 so that it forms a vertical angle with ∠1.



c. Name all vertical angle pairs in the diagram shown.



1 + 2

3 + 4

d. Measure each angle in part (c). What do you notice?

Vertical angles are always congruent

Vertical angles are two nonadjacent angles that are formed by two intersecting lines, line segments, or rays.

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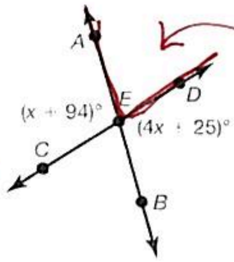
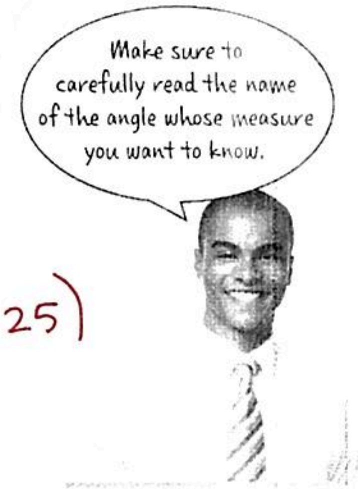
4. Determine $m\angle AED$. Explain how you determined the angle measure.



$180 - 117 =$ $\boxed{63^\circ}$



$x + 94$

$23 + 94$

$\boxed{117^\circ}$

$(x + 94) = (4x + 25)$

$69 = 3x$

$23 = x$

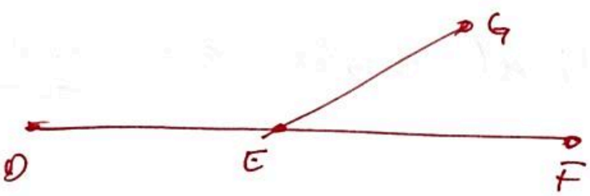
5. For each conditional statement, draw a diagram and then write the hypothesis as the "Given" and the conclusion as the "Prove."

a. $m\angle DEG + m\angle GEF = 180°$, if $\angle DEG$ and $\angle GEF$ are a linear pair.

Given: $\angle DEG$ ad $\angle GEF$ are linear pair

Prove:

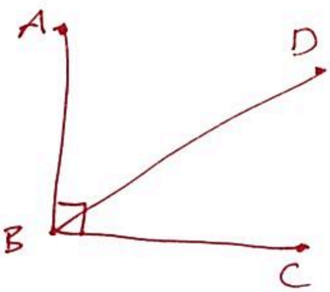
$\angle DEG + \angle GEF = 180°$



b. If $\angle ABD$ and $\angle DBC$ are complementary, then $\overrightarrow{BA} \perp \overrightarrow{BC}$.

Given: $\angle ABD$ and $\angle DBC$ are complementary

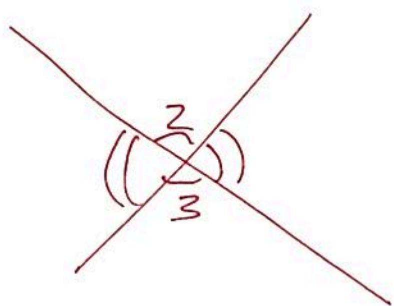
Prove: $\overrightarrow{BA} \perp \overrightarrow{BC}$



c. If $\angle 2$ and $\angle 3$ are vertical angles, then $\angle 2 \cong \angle 3$.

Given: $\angle 2$ and $\angle 3$ vertical angles

Prove: $\angle 2 \cong \angle 3$



PROBLEM 3 Postulates and Theorems

A **postulate** is a statement that is accepted without proof.

A **theorem** is a statement that can be proven.

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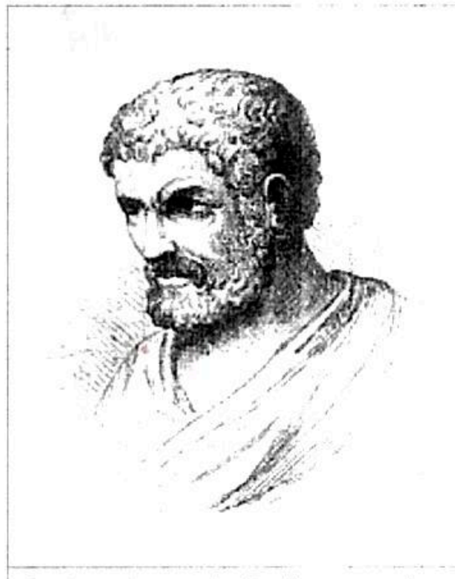
The Elements is a book written by the Greek mathematician Euclid. He used a small number of undefined terms and postulates to systematically prove many theorems. As a result, Euclid was able to develop a complete system we now know as **Euclidean geometry**.



Greek mathematician Euclid is sometimes referred to as the Father of Geometry.

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Euclid's first five postulates are:

1. A straight line segment can be drawn joining any two points.
2. Any straight line segment can be extended indefinitely in a straight line.
3. Given any straight line segment, a circle can be drawn that has the segment as its radius and one endpoint as center.
4. All right angles are congruent.
5. If two lines are drawn that intersect a third line in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough. (This postulate is equivalent to what is known as the parallel postulate.)

Euclid used only the first four postulates to prove the first 28 propositions or theorems of *The Elements*, but was forced to use the fifth postulate, the parallel postulate, to prove the 29th theorem.

The *Elements* also includes five "common notions":

1. Things that equal the same thing also equal one another.
2. If equals are added to equals, then the wholes are equal.
3. If equals are subtracted from equals, then the remainders are equal.
4. Things that coincide with one another equal one another.
5. The whole is greater than the part.

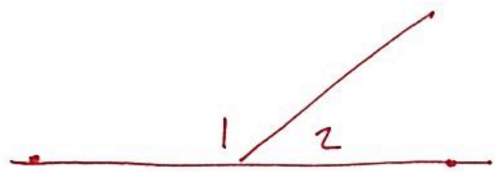
It is important to note that Euclidean geometry is not the only system of geometry. Examples of non-Euclidian geometries include hyperbolic and elliptic geometry. The essential difference between Euclidean and non-Euclidean geometry is the nature of parallel lines.

You will use these postulates to make various conjectures. If you are able to prove your conjectures, then the conjectures will become theorems. These theorems can then be used to make even more conjectures, which may also become theorems. Mathematicians use this process to create new mathematical ideas.

The **Linear Pair Postulate** states: "If two angles form a linear pair, then the angles are supplementary."

8. Use the Linear Pair Postulate to complete each representation.

a. Sketch and label a linear pair.



b. Use your sketch and the Linear Pair Postulate to write the hypothesis.

$\angle 1$ and $\angle 2$ form a linear pair

c. Use your sketch and the Linear Pair Postulate to write the conclusion.

$\angle 1$ and $\angle 2$ are supplementary

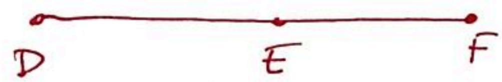
d. Use your conclusion and the definition of supplementary angles to write a statement about the angles in your figure.

$m\angle 1 + m\angle 2 = 180^\circ$

The **Segment Addition Postulate** states: "If point B is on $\overline{AC}$ and between points A and C, then $AB + BC = AC$."

9. Use the Segment Addition Postulate to complete each representation.

a. Sketch and label collinear points D, E, and F with point E between points D and F.



b. Use your sketch and the Segment Addition Postulate to write the hypothesis.

Point E is on $\overline{DF}$ and between points D and F

c. Use your sketch and the Segment Addition Postulate to write the conclusion.

$DE + EF = DF$

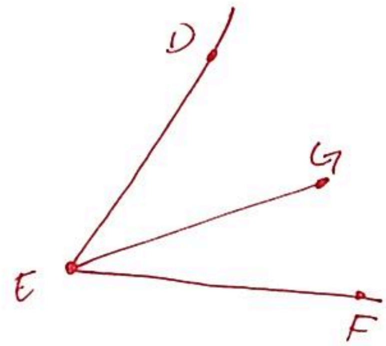
d. Write your conclusion using measure notation.

$m\overline{DE} + m\overline{EF} = m\overline{DF}$

The **Angle Addition Postulate** states: "If point D lies in the interior of $\angle ABC$, then $m\angle ABD + m\angle DBC = m\angle ABC$."

10. Use the Angle Addition Postulate to complete each representation.

a. Sketch and label $\angle DEF$ with $\overrightarrow{EG}$ drawn in the interior of $\angle DEF$.



b. Use your sketch and the Angle Addition Postulate to write the hypothesis.

Point D lies on the interior of $\angle DEF$

c. Use your sketch and the Angle Addition Postulate to write the conclusion.

$m\angle DEG + m\angle GEF = m\angle DEF$

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Be prepared to share your solutions and methods.