# Transforming to a New Level!

# 3.1

## Using Transformations to Determine Area

### LEARNING GOALS

In this lesson, you will:

- Determine the areas of squares on a coordinate plane.
- Connect transformations of geometric figures with number sense and operations.
- Determine the areas of rectangles using transformations.
- Determine and describe how proportional and non-proportional changes in the linear dimensions of a rectangle affect its perimeter and area.

You've probably been in a restaurant or another public building and seen a sign like this:

| |
|---|
| **MAXIMUM OCCUPANCY**<br>480 |

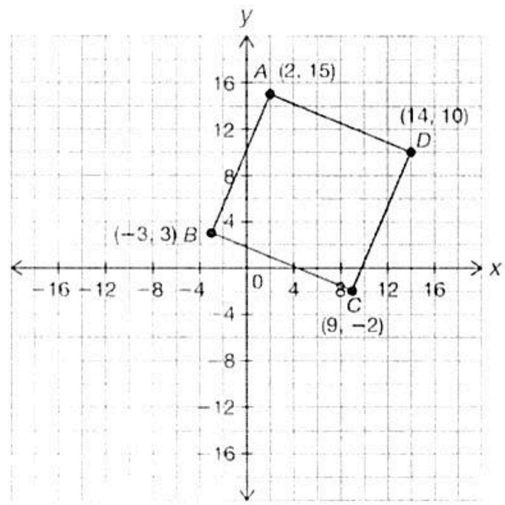
What does this mean? It means that the maximum number of people that can be in that space cannot—by law—be more than 480.

Why does this matter? Well, occupancy laws are often determined by the fire marshal of a town or city. If too many people are in a space when an emergency occurs, then getting out would be extremely difficult or impossible with everyone rushing for the exits. So, occupancy laws are there to protect people in case of emergencies like fires.

Of course, the area of a space is considered when determining maximum occupancy. Can you think of other factors that should be considered?

## PROBLEM 1 Pomp and Circumstance

Marissa is throwing a party for her graduation and wants to invite all of her friends and their families. Consider the space defined by quadrilateral *ABCD*. Each of the four corners of the space is labeled with coordinates, measured in feet, and defines the dimensions of the room that Marissa's little brother says the party should be held.




1. Marissa's mom says that the room is obviously a square or a rectangle, so if you can figure out the length of one or two of the sides, then you can easily determine the area.

   Marissa tells her mother that you can't just assume that a shape is a square or a rectangle because it looks like one.

   Who is correct and why?

2. List the properties of each shape.

a. squares

- quadrilateral (4 sided polygon)
- 4 congruent sides
- 4 right angles

b. non-square rectangles

- quadrilateral (4 sided polygon)
- 2 pairs of parellel sides
- each pair of sides ≅
- 4 right angles

3. How can you use the properties you listed in Question 2 to determine whether the room is a square or a non-square rectangle?

find the side lengths to determine if they're congruent.

4. A rule of thumb for determining the maximum occupancy of a room is that each person in the room is given 36 square feet of space.

Predict the maximum occupancy of the room Marissa wants to rent. Describe the information you need and the strategies you could use to improve your prediction.

- find area of room by finding at least two side lengths

- Divide area by 36 to determine max occupancy

$*$ Slope $\frac{\text{Change of } y}{\text{change of } x} = \frac{y_2 - y_1}{x_2 - x_1}$

$*$ distance Formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

5. Determine if quadrilateral *ABCD* is a square or a rectangle. Show your work.

$AB = \sqrt{(2-(-3))^2 + (15-3)^2}$
$= \sqrt{5^2 + 12^2}$
$= \sqrt{25 + 144}$
$= \sqrt{169}$
$= 13$

Slope AB
$\frac{15-3}{2-(-3)} = \frac{12}{5}$

$CD = \sqrt{(14-9)^2 + (10-(-2))^2}$
$= \sqrt{5^2 + 12^2}$
$= \sqrt{25 + 144}$
$= \sqrt{169}$
$= 13$

Slope CD
$\frac{-2-10}{9-14} = \frac{12}{5}$

$BC = \sqrt{(-3-9)^2 + (3-(-2))^2}$
$= \sqrt{-12^2 + 5^2}$
$= \sqrt{144 + 25}$
$= \sqrt{169}$
$= 13$

Slope BC
$\frac{3-(-2)}{-3-9} = -\frac{5}{12}$

$AD = \sqrt{(14-2)^2 + (10-15)^2}$
$= \sqrt{12^2 + 5^2}$
$= \sqrt{144 + 25}$
$= \sqrt{169}$
$= 13$

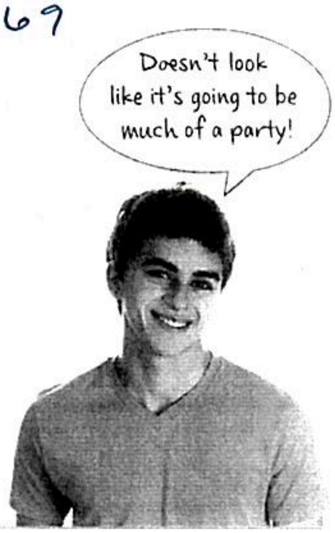
Slope AD
$\frac{10-15}{14-2} = -\frac{5}{12}$

6. Use the rule of thumb from Question 4 to determine the maximum number of people that Marissa can invite to her party.

Slopes are $\perp \rightarrow$ Square
Lines are $\cong \rightarrow$

Area = L · w
= 13 · 13
= 169

$169 \div 36 \approx 4.69$



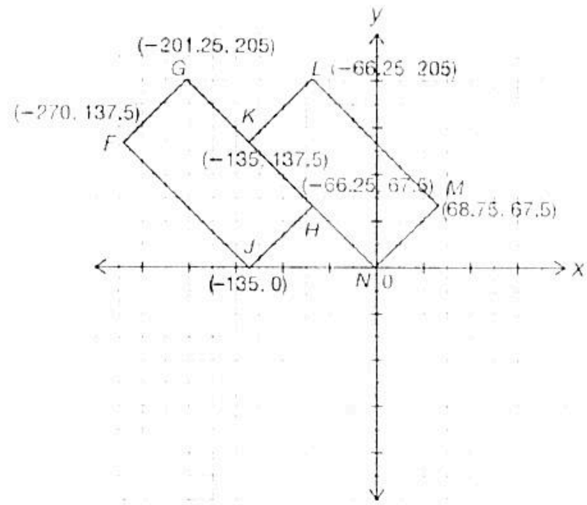

7. Do you think this location is reasonable for Marissa's graduation party? Why or why not?

NO, She'd only be able to invite about 3 people.

## PROBLEM 2 Maximum Efficiency

In addition to formulas and properties, you can also use transformations to help make your problem solving more efficient.

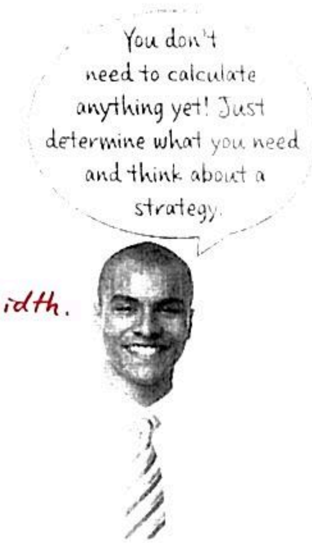
The figure shown on the coordinate plane is composed of two quadrilaterals, *FGHJ* and *KLMN*.




1. Describe the information you need and the strategies you can use to determine the total area of the figure.

Need to determine x's and sides to find shape

then find Area by Length x width.



2. Colby says that the two quadrilaterals are congruent. He says that knowing this can help him determine the area of the figure more efficiently. Is Colby correct? Explain your reasoning.

If they are ≅, he would only need to find area of one figure.

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3. Describe a transformation you can use to determine whether the two quadrilaterals are congruent. Explain why this transformation can prove congruency.


Translate each vertice of one of the figures, if the coordinates end up as same as other figure, they are ≅

4. Apply the transformation you described in Question 3 to determine if the quadrilaterals are congruent. Show your work and explain your reasoning.

y-coordinates already the same.

translate Quad. GFJH to the right 135 units.

5. Tomas had an idea for solving the problem even more efficiently.

Skip

Tomas

When a polygon has vertices that are on the $x$- or $y$-axis or are at the origin, it is a little easier to use the Distance Formula, because one or more of the coordinates are 0.

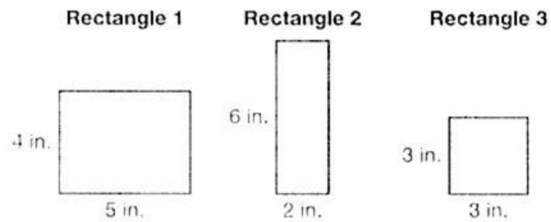
Explain why Tomas is correct.

6. Which quadrilateral would Tomas choose and why? Determine the area of the entire figure.

## PROBLEM 3 Reach for New Heights and Bases

When the dimensions of a two-dimensional figure change, its perimeter and area change as well. Let's investigate how both proportional and non-proportional changes in a figure's dimensions affect its perimeter and area.

1. Consider the following rectangles with the dimensions shown.

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Complete the table to determine how doubling or tripling each rectangle's base and height affects its perimeter and area. The information for Rectangle 1 has been done for you.

| | | Original Rectangle | Rectangle Formed by Doubling Dimensions | Rectangle Formed by Tripling Dimensions |
|---|---|---|---|---|
| | **Linear Dimensions** | $b = 5$ in.<br>$h = 4$ in. | $b = 10$ in.<br>$h = 8$ in. | $b = 15$ in.<br>$h = 12$ in. |
| **Rectangle 1** | **Perimeter (in.)** | $2(5 + 4) = 18$ | $2(10 + 8) = 36$ | $2(15 + 12) = 54$ |
| | **Area (in.$^2$)** | $5(4) = 20$ | $10(8) = 80$ | $15(12) = 180$ |
| | **Linear Dimensions** | b = 6 in<br>h = 2 in | b = 12 in<br>h = 4 in | b = 18 in<br>h = 6 in |
| **Rectangle 2** | **Perimeter (in.)** | 16 in | 32 in | 48 in |
| | **Area (in.$^2$)** | 12 in | 48 in | 108 in |
| | **Linear Dimensions** | b = 3 in<br>h = 3 in | b = 6 in<br>h = 6 in | b = 9 in<br>h = 9 in |
| **Rectangle 3** | **Perimeter (in.)** | 12 in | 24 in | 36 in |
| | **Area (in.$^2$)** | 9 in | 36 in | 81 in |

2. Describe how a proportional change in the linear dimensions of a rectangle affects its perimeter.


   a. What happens to the perimeter of a rectangle when its dimensions increase by a factor of 2?

   2 → perimeter increases by factor of 2

   b. What happens to the perimeter of a rectangle when its dimensions increase by a factor of 3?

   3 → $\times 3$

   c. What would happen to the perimeter of a rectangle when its dimensions increase by a factor of 4?

   4 → $\times 4$

   d. Describe how you think the perimeter of the resulting rectangle would compare to the perimeter of a $4 \times 10$ rectangle if the dimensions of the original rectangle were reduced by a factor of $\frac{1}{2}$. Then, determine the perimeter of the resulting rectangle.

   would be ½ the perimeter...
   original perimeter = 28
   $4 * \frac{1}{2} = 2$
   $10 * \frac{1}{2} = 5$

   new perimeter = 14

   e. In terms of $k$, can you generalize change in the perimeter of a rectangle with base $b$ and height $h$, given that its original dimensions are multiplied by a factor $k$?

3. Describe how a proportional change in the linear dimensions of a rectangle affects its area.

a. What happens to the area of a rectangle when its dimensions increase by a factor of 2?

$2^2 \rightarrow$ 4 times greater

b. What happens to the area of a rectangle when its dimensions increase by a factor of 3?

$3^2 \rightarrow$ 9 times greater

c. What would happen to the area of a rectangle when its dimensions increase by a factor of 4?

$4^2 \rightarrow$ 16 times greater

d. Describe how you think the area of the resulting rectangle would compare to the area of a 4 × 10 rectangle if the dimensions of the original rectangle were reduced by a factor of $\frac{1}{2}$. Then, determine the area of the resulting rectangle.

$\frac{1}{2^2} \rightarrow \frac{1}{4}$ as big

$4 \times \frac{1}{2} = 2$ original area = 40

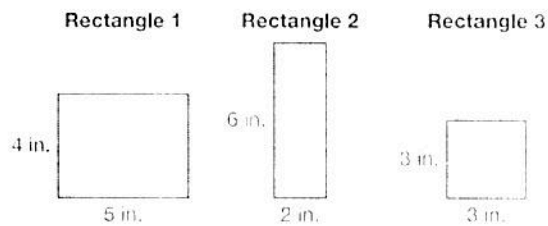
$10 \times \frac{1}{2} = 5$ new area = 10

e. In terms of $k$, can you generalize change in the area of a rectangle with base $b$ and height $h$, given that its original dimensions are multiplied by a factor $k$?

Non-proportional change to linear dimensions of a two-dimensional figure involves adding or subtracting from the side lengths.

4. Do you think a non-proportional change in the linear dimensions of a two-dimensional figure will have the same effect on perimeter and area as proportional change? Explain your reasoning.

5. Consider the following rectangles with the dimensions shown.




Complete the table to determine how adding two or three inches to each rectangle's base and height affects its perimeter and area. The information for Rectangle 1 has been done for you.

| | | Original Rectangle | Rectangle Formed by Adding 2 Inches to Dimensions | Rectangle Formed by Adding 3 Inches to Dimensions |
|---|---|---|---|---|
| **Rectangle 1** | **Linear Dimensions** | $b = 5$ in.<br>$h = 4$ in. | $b = 7$ in.<br>$h = 6$ in. | $b = 8$ in.<br>$h =$ [illegible] in. |
| | **Perimeter (in.)** | $2(5 + 4) = 18$ | $2(7 + 6) = 26$ | $2(8 + 7) =$ [illegible] |
| | **Area (in.²)** | $5(4) = 20$ | $7(6) = 42$ | $8(7) = 56$ |
| **Rectangle 2** | **Linear Dimensions** | b = 6<br>h = 2 | b = 8<br>h = 4 | b = 9<br>h = 5 |
| | **Perimeter (in.)** | 16 in | 24 in | 28 in |
| | ~~**Area (in.²)**~~ | | | |
| **Rectangle 3** | **Linear Dimensions** | b = 3<br>h = 3 | b = 5<br>h = 5 | b = 6<br>h = 6 |
| | **Perimeter (in.)** | 12 in | 20 in | 24 in |
| | ~~**Area (in.²)**~~ | | | |

6.) non-proportional change in linear dimensions of a rectangle.

a.). when add/subtract from all 4 sides
- multiply by 4 and change perimeter by that amount.

Add 2 to each side → 2(4) → perimeter increases by 8

Add 3 to each side → 3(4) → perimeter increases by 12

Add 4 to each side → 4(4) perimeter will increase by 16

Subtract 2 from each side → 2(4) perimeter will decrease by 8

7.) There is not a clear pattern to determine change of area when linear dimensions are change by a non-proportional amount.